On the Feasibility of Equating Tax to Economic Depreciation

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In *Blueprints for Basic Tax Reform* (January 1977) the Treasury proposed, as part of its income-based tax plan, that tax depreciation allowances be set approximately equal to real economic depreciation of the capital stock. For some time now a number of well-known economists, e.g., Samuelson (1964), Eisner (1973), Coen (1975), and Break (1974), but by no means all, have been recommending that tax depreciation be brought into line with real economic depreciation.[1] Because economic depreciation is difficult to measure, economists, in order to aid in the assessment of such a proposal, should consider its feasibility and implementation, as well as its consequences for equity, efficiency, stabilization, and growth.

Economists have, of course, devoted considerable energy to studying the consequences of the corporate income tax system, including the effects of changes in parameters such as deductions for depreciation. Break (1974) has analyzed recent progress in this field and identified the major unresolved issues. Considerable professional disagreement as to the precise effects of changing deductions remains. Nevertheless, judging from the Treasury's *Blueprints*, realignment of tax deductions for depreciation accord-

[1] For an alternative point of view, see Smith (1963).

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ing to the economic depreciation norm seems to be a serious possibility. Therefore, we shall take the proposal as given and shall address ourselves to two pressing questions we believe have received inadequate attention. First, is the proposal feasible? Specifically, is it possible to measure, with sufficient precision, the patterns and rates of economic depreciation to which tax deductions are to conform so that the proposal can be implemented? Second, what types of patterns should be adopted so that tax deductions approximate economic patterns—straight line, decelerated, or accelerated?

Many economists seem skeptical about the feasibility of measuring economic depreciation of many classes of assets. The belief that used-asset prices are not available because most assets are rarely marketed seems to be widespread.[2] Coen (1975), for example, argues:

> Economists have attempted to measure economic depreciation principally by examining relationships between market prices of certain types of new and used assets—farm tractors, autos, and pick-up trucks, for example, but the absence of active markets for many types of used industrial capital goods greatly limits opportunities for this type of analysis. (p. 59)

Discussion at the Department of the Treasury 1975 Conference on Tax Research revealed some doubt about the general applicability of depreciation studies based on vintage price data.[3] Shoven and Bulow (1975) also question the practicality of measuring depreciation.

Undoubtedly many assets are rarely traded when used; however, we believe many are traded fairly often. Vigorous used-asset markets exist within and across numerous industries, between private and public institutions, and even between advanced and less developed countries. Recently, for example, the Office of Industrial Economics (OIE) produced a business buildings survey covering

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[2] The dependence of economic depreciation on tax depreciation is a closely related issue raised by Feldstein and Rothschild (1974) and by Hulten and Wykoff (1976). Any change in the tax code will, in general, change asset prices and thus change economic depreciation. This dependence suggests that, *as a practical matter,* the two types of depreciation must be brought into line by a sequence of tax code changes. The measurement of economic depreciation (the topic of this paper) would, in principle, be required at each policy iteration.

[3] See U.S. Department of the Treasury, Office of Tax Analysis, *Conference on Tax Research 1975.*

more than 15 classes of commercial and industrial structures and containing vintage asset prices.[4] Furthermore, numerous equipment categories, in addition to vehicles, are traded in secondhand markets: machine tools, construction equipment, and office furniture and machinery. While this list is by no means exhaustive, it is sufficiently diverse to suggest that further studies along the lines of the OIE survey are possible.[5]

Given the availability of used-asset price data—both actual and potential—we believe that it is useful to investigate the problems involved in estimating economic depreciation from used-asset prices. In the next section of this paper we sketch briefly our general method for estimating depreciation patterns and rates and apply this method to four large classes of commercial and industrial structures: factories, office buildings, retail trade stores, and warehouses. This section summarizes research reported in more detail elsewhere. Although much remains to be done, we believe this approach is worth pursuing. It may be applied to either vintage acquisition or rental prices, and, while some assets are neither resold when used nor rented, many others are marketed one way or another.

Our principal econometric results are (a) that economic depreciation of the structures studied is accelerated vis-à-vis straight-line, and (b) that the rate of economic depreciation is about half that allowed under the tax code. The second result is not a major surprise. The first, however, may be. Although little actual empirical research has been done in this area, we have encountered considerable professional opinion that buildings depreciate as "one-hoss shays," i.e., no physical in-place deterioration, merely a decline in value due to the approach of retirement. The age-price curve of a "one-hoss-shay" asset would be concave, revealing a

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[4] In 1975, commercial and industrial structures accounted for 40 percent of the purchases of new private nonresidential structures, and for 14 percent of all private nonresidential fixed investment. See tables 1.1, 5.4, and 5.6 of the U.S. National Income and Product Accounts, as reported in *Survey of Current Business* (July 1976).

[5] The Asset Depreciation Range information system must also be mentioned as a potentially invaluable source of data. As currently structured, the ADR vintage accounts are highly aggregated and are more suited to imputing retirement profiles than to providing data on used-asset values. If the ADR system is extended to cover all business taxpayers, as suggested in the Treasury's *Blueprints for Basic Tax Reform* (1977), and is appropriately modified, it could ultimately yield data suitable for estimating economic depreciation.

pattern decelerated vis-à-vis straight-line.[6] In their study of office buildings, Taubman and Rasche (1969) actually report such a concave function. In 1975, Coen reported that ". . . structures in the majority of industries suffer no loss in productive capacity over their service lives (they resemble one-hoss shays)." However, in more recent work, Coen (1976) has found that structures appear to depreciate in a convex, or accelerated, pattern.

Since our results run counter to the conventional wisdom, we illustrate in a later section the underlying data for the large class of office buildings and show that the age-price curve for this data is convex. We also analyze the office building class in greater detail than in our previous studies. Specifically, our earlier results were derived from pooled data on buildings that were purchased at different points in time. It is possible that our estimation procedures did not adequately isolate the process of aging from other processes associated with the passing of time (inflation, tax code changes, etc.). In order to investigate this possibility, we break the office building data into a series of cross sections and analyze each cross section separately. The results for the separate cross sections are roughly consistent with the results of the pooled analysis, and this new evidence indicates that our treatment of the date of purchase with the pooled data does not alter our conclusions. In a final section we compare our results with those obtained by Taubman and Rasche (1969) and Coen (1975; 1976).

The implications of our analysis for tax reform may be briefly summarized. We are able to provide tentative conclusions about the rate and form of economic depreciation for commercial and industrial structures. Accelerated patterns, such as those currently allowed by the Federal Tax Code, are warranted; however, the rates of depreciation, implied by the tax lives, appear to be too

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[6] To see that an asset with a "one-hoss-shay" efficiency pattern has a concave age-price curve, we note that the "one-hoss-shay" maintains a constant level of efficiency until it is retired. Thus, under the perfect foresight model considered in this paper

$$Y(t)=\int_{x=t}^{T} \overline{C}\, e^{-r(T-x)}dx=\frac{\overline{C}}{r}[1-e^{-r(T-t)}],$$

where $Y(t)$ is the asset's price, C is the (constant) rental price of a new asset, and r is the (constant) rate of discount. Differentiating, $\frac{dY}{dt}=-\overline{C}\, e^{-r(T-t)}<0$ (dY/dt is the slope of the age-price curve, which is everywhere negative). Furthermore, $\frac{d^2Y}{dt^2}=-r\,\overline{C}\, e^{-r(T-t)}<0$, implying that the age-price curve is uniformly concave.

rapid. We note that these conclusions are tentative because they depend upon only one sample of vintage price data and because, as will be seen later, adequate data on asset retirement is not available.

On the Theory and Measurement of Economic Depreciation

Theoretical Framework

We define economic depreciation as the fall in an asset's price as it ages. Following Hotelling (1925), Hall (1968), and Jorgenson (1974), we assume efficient and competitive capital markets in which, under perfect certainty, the acquisition price of an asset will equal the present discounted value of the future flow of after-tax user costs, inclusive of tax credits and depreciation allowances, up to retirement plus the present value of any retirement value of the asset. Departing from the Hotelling-Hall-Jorgenson model, we focus on the average price of an entire cohort, or vintage, of homogeneous assets rather than on one single asset. While each asset in the cohort is assumed to be identical while in place, each has a different, yet certain, retirement date. For purposes of estimating average cohort depreciation, the relevant price, or rental, is that of the average asset in the original cohort. For example, if all *N* new assets in a cohort were pooled to form a company in which *N* stock certificates were issued, then the average price would be that of one certificate, and the average rental would be the average annual return on a share.

When dealing with used assets, only the prices of survivors are usually available; yet to study the vintage performance of the original cohort one must study the average used price both of survivors and of retired assets. Under our assumptions, the average cohort used price can be shown to be equivalent to the weighted sum of the average price of survivors and that of nonsurvivors, where the weights are the probabilities of having survived and not survived, respectively. Assuming, as we do, that retired assets are worthless, calculation of the average cohort used price merely corresponds to premultiplying observed vintage prices by their probability of having survived to that age. Some of these assumptions may be relaxed, but, in light of our data, it is pointless to relax them.

Even though in this paper we deal with purchase prices, the fact that the same theoretical framework applies to the analysis of rental prices must be emphasized. The relevant average cohort rental prices are simply the observed average used rentals premultiplied by their survival probability. The rental, under our assumptions, is equivalent to the sum of the rate of return plus depreciation minus revaluation, all calculated on the weighted average cohort used prices. Thus, this theory is perfectly general in the sense that it may be applied to structures or machinery, rental data or purchase data.

Data on Commercial and Industrial Structures

The preceding theoretical framework is used to estimate the economic depreciation of 16 classes of commercial and industrial structures from the vintage acquisition prices reported by owners to the Internal Revenue Service's Office of Industrial Economics.[7] The sample, consisting of 8,066 observations taken in 1972, contains reports by current owners of the original prices they paid, net of land, to acquire the buildings. About one-third of the buildings were used when purchased, so the acquisition prices per square foot can be indexed by age of structure and date of purchase. In other words, the data can be regarded as a rectangular array, by age and date, of the average acquisition prices per square foot in each class of structures.

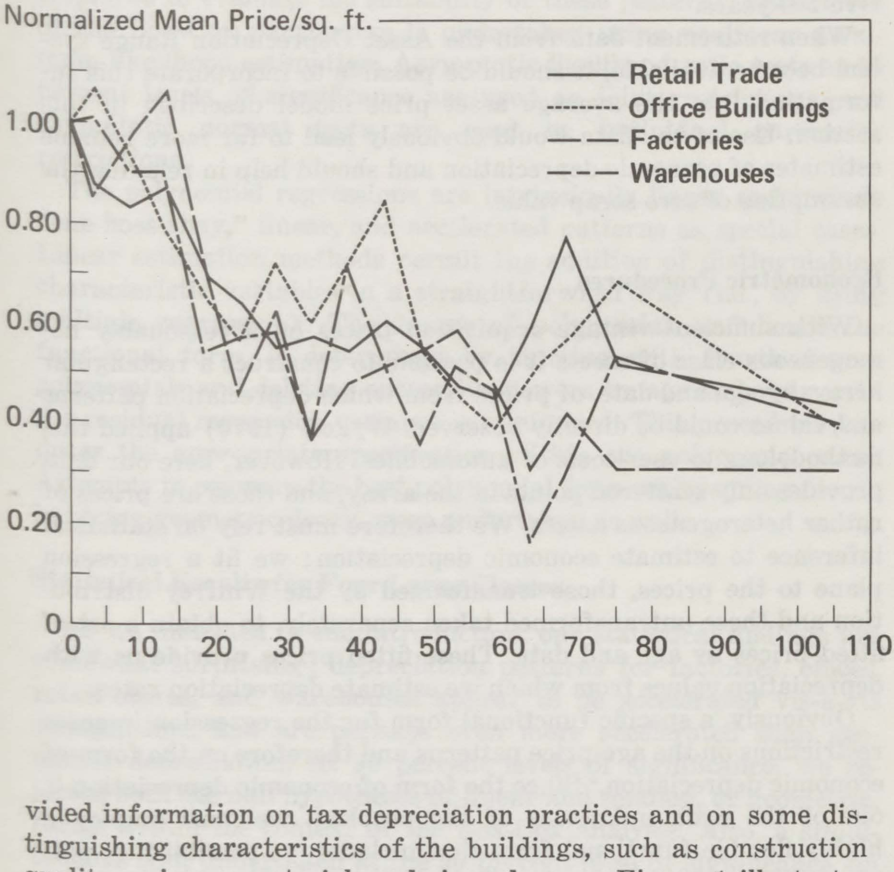
We report here on our analysis of the four largest classes of structures: factories, office buildings, retail trade stores, and warehouses. Table 1 contains summary statistics on these classes, illustrating the richness of the data. Each observation also pro-

[7] The survey results are reported by the U.S. Department of the Treasury, Office of Industrial Economics (1975). In our research we used unweighted price data because the sampling weights employed by the Office of Industrial Economics were too variable and appeared to lead to the risk of overemphasizing individual observations. Details are explained in Hulten and Wykoff (1977).

TABLE 1.—*Summary statistics by asset class*

Class	Number of observations	Number used	Price/sq.ft. Mean	Price/sq.ft. SD	Age (years) Mean	Age (years) SD	*Bulletin F* mean life
Large Classes							
Retail trade	1666	1013	7.31	7.89	20.50	24.22	67
Offices	1654	889	12.20	9.80	16.68	23.84	67
Warehouse	580	275	5.20	5.10	11.77	19.66	75
Factory	526	282	5.60	5.92	15.84	22.14	80

FIGURE 1.—*Mean asset price by age interval untrans/undefl.*



vided information on tax depreciation practices and on some distinguishing characteristics of the buildings, such as construction quality, primary material, and zip code area. Figure 1 illustrates the general age-price pattern of the raw data for each of these large classes. It depicts the average acquisition price per square foot in each 5-year age interval against age with no adjustments for retirement or date of purchase.

Unfortunately, at this time we do not have actual data on the retirement distribution of structures. We therefore apply a Winfrey L_0 retirement distribution [8] to the mean life, as approximated in *Bulletin F*, U.S. Department of the Treasury (1942). While the choice of the L_0 distribution is not based on hard evidence, it has the virtue of allowing for the fairly gradual retirement of assets with the possibility that some structures survive to very old ages.

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[8] See Winfrey (1935).

To illustrate, if a cohort's mean asset life is 50 years, then, according to the L_0 distribution, 92 percent survive 10 years, 69 percent survive 30 years, 45 percent survive 50 years, and 8 percent survive 100 years.

When retirement data from the Asset Depreciation Range system become available, it should be possible to incorporate this information into the average asset price model described in this section. Use of the data would obviously lead to far more reliable estimates of economic depreciation and should help in relaxing the assumption of zero scrap value.

Econometric Procedures

With sufficient vintage acquisition prices on a reasonably homogeneous class of assets it is possible to construct a rectangular array, by age and date, of prices from which depreciation patterns and values could be directly observed. Wykoff (1970) applied this methodology to user-costs of automobiles. However, here our data provides only scattered points in the array, and those are prices of rather heterogeneous assets. We therefore must rely on statistical inference to estimate economic depreciation: we fit a regression plane to the prices, those transformed by the Winfrey distribution and those untransformed taken separately, to obtain a set of fitted prices by age and date. These fitted prices provide us with depreciation values from which we estimate depreciation rates.

Obviously, a specific functional form for the regression imposes restrictions on the age-price patterns and therefore on the form of economic depreciation.[9] Since the form of economic depreciation is one of the main unknowns of the problem, we must employ a highly flexible functional form that includes, as possibilities, patterns such as "one-hoss shay," straight-line, and accelerated depreciation. In our analysis, we used two such specifications: the Box-Cox power transformation and the polynomial regression.

The Box-Cox power transformation, an intrinsically nonlinear procedure discussed at some length by Zarembka (1974), permits joint estimation of (a) parameters that determine a specific functional form within the Box-Cox class and (b) parameters that

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[9] For example, the "one-hoss-shay" age-price pattern is given in footnote 8, and the corresponding rate of depreciation is given by the $\frac{d(\ln Y)}{dt}$. Since $\overline{C}$ and r are constant, this derivative is equivalent to the change in price as the asset ages. The Box-Cox age-price pattern is given by equation (1) of the third section of this paper, and the corresponding rate of depreciation can be calculated from equation (3).

determine the slope and intercept. Since certain restrictions on the unknown form parameters produce "one-hoss-shay," linear, and geometric forms, classical hypothesis-testing procedures may be employed to evaluate the suitability of these patterns. Estimation of the Box-Cox parameters is undertaken using nonlinear maximum likelihood estimation. Asymptotic likelihood ratio tests at 95 percent levels of significance are used on joint restrictions, and asymptotic normal tests are used on individual parameter restrictions.

The polynomial regressions are intrinsically linear and include "one-hoss-shay," linear, and accelerated patterns as special cases. Linear estimation methods permit the addition of distinguishing characteristic variables in a straightforward way (i.e., by using multiple regression). The degree of polynomial, and hence the functional form, is determined by starting with fourth degree polynomials and deleting successive powers by age and year until the residual regression variance is minimized. This procedure produces the appropriate specification within the polynomial class. Attempts to compare the best polynomial form with semilog forms, implying geometric decay, were undertaken as well.

Statistical Results for Four Large Classes

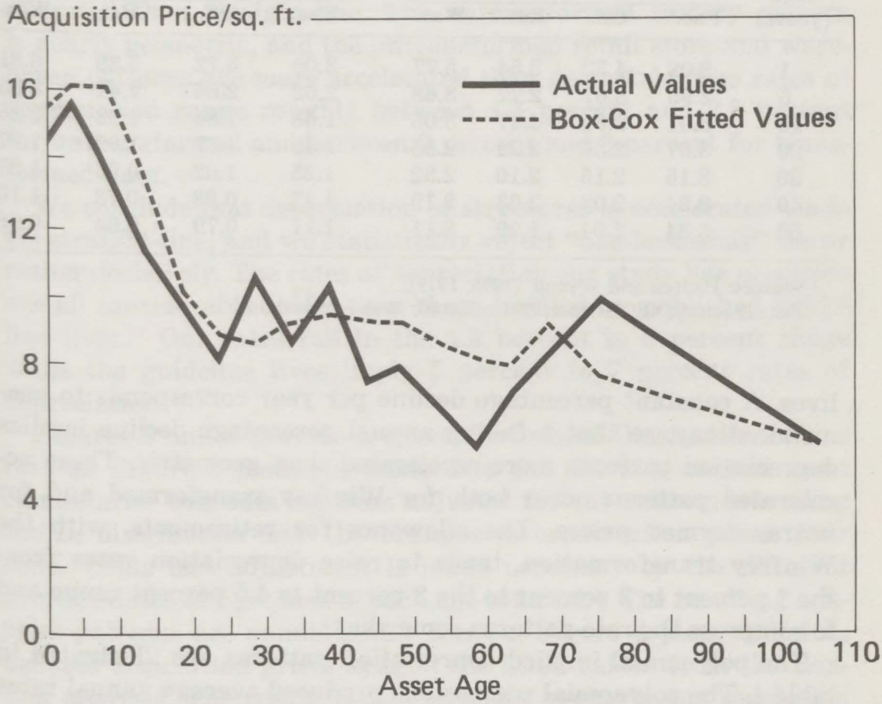
As we indicated in the introduction, our statistical findings are somewhat surprising: depreciation patterns for factories, offices, retail stores, and warehouses appear to be accelerated vis-à-vis straight-line, and are perhaps even more accelerated than geometric depreciation. At 95 percent levels of significance, we rejected both the null hypotheses of linear and semilog, or geometric, forms within the context of the Box-Cox analysis. Also, a simultaneous test, undertaken at the 95 percent level of significance, on two form parameters only, corresponding to geometric depreciation (a weaker restriction than the semilog constraint), indicates rejection in all four classes of the geometric pattern.

The polynomial tests lead to similar results. Table 2 contains the forms of the polynomial price equations. The most common forms in the age variable were convex, bowl-shaped, quadratic equations and cubic equations that were relatively flat convex forms over most of the range of observed ages. When modified sums of squared residuals from the semilog equations were compared with the best polynomial sums of squares and vice versa, a procedure suggested by Theil (1971), the results were inconclusive. The null hypothesis of linear depreciation and of a concave age-price curve were rejected at the 95 percent level of significance.

TABLE 2.—*Polynomial price equations*

Class	L_0–Transformed			Untransformed			Inc/Pop	No. of Dummies
	Age (s)	Year (t)	$\bar{R}^2$	Age (s)	Year (t)	$\bar{R}^2$		
Factory	Cubic	Linear	.22	Cubic	Linear	.10	Y/N	1
Offices	Cubic	Quadratic	.34	Cubic	Quadratic	.20	Y	2
Retail trade	Quadratic	Cubic	.24	Cubic	Cubic	.16	$Y/N,N$	1
Warehouse	Quadratic	Linear	.16	Cubic	Linear	.11	Y/N	1

FIGURE 2.—*Office building: fitted vs. actual price by age interval untrans.*



The Box-Cox and polynomial point estimates for one class—office buildings—are given in the appendix (tables A–1 and A–2).[10] Figures 2 and 4 summarize the results by 5-year age intervals. It is evident from these figures that (a) the fitted values of the dependent variable (acquisition price per square foot) approximate the actual value rather well, and (b) that the fitted age-price pattern is convex. The convex age-price pattern is clearly evident in the underlying data and cannot be attributed to our choice of statistical techniques.

Specific average annual rates of economic depreciation are implied by each of the forms estimated. Table 3 contains some select annual percentage decline rates from the Box-Cox analysis.

The most striking feature of the table 3 rates is that they are largest for young buildings and tend to fall throughout assets'

[10] Office buildings were selected for detailed analysis for reasons that will become clear in the next section of this paper. Details are given by Hulten and Wykoff (1975; 1976; 1977).

TABLE 3.—*Selected rates of depreciation from Box-Cox analysis* [1]
(*percentage decline per year*)

Age	Class (transformed)				Class (untransformed)			
(years)	Fac.[2]	Off.	Ret.	War.	Fac.	Off.	Ret.	War.
1	3.02	4.32	3.54	5.77	3.00	5.72	5.39	6.81
5	2.99	3.07	2.77	3.68	2.02	2.66	2.41	3.23
10	3.01	2.64	2.47	3.05	1.68	1.84	1.63	2.26
20	3.07	2.30	2.22	2.55	1.39	1.27	1.09	1.57
30	3.15	2.15	2.10	2.32	1.25	1.02	0.86	1.27
40	3.24	2.08	2.03	2.19	1.17	0.88	0.73	1.10
50	3.34	2.04	1.99	2.11	1.11	0.79	0.64	0.98

[1] Sources: Hulten and Wykoff (1976, 1977).
[2] Fac.=Factory; Off.=Office; Ret.=Retail; War.=Warehouse

lives. A constant percentage decline per year corresponds to geometric decay, so that a falling annual percentage decline implies depreciation patterns more accelerated than geometric. These accelerated patterns occur both for Winfrey transformed and for untransformed prices. The allowance for retirements, with the Winfrey transformation, tends to raise depreciation rates from the 1 percent to 3 percent to the 3 percent to 4.5 percent range and to compress the rate patterns somewhat.

The polynomial implied depreciation patterns are illustrated in table 4. The polynomial regressions produced average annual rates of decline that are both smaller and less variable than those of the Box-Cox analysis. Furthermore, these rates tend to rise, though quite gradually for the transformed data, over asset life. We cal-

TABLE 4.—*Select rates of depreciation from polynomial analysis* [1]
(*percentage decline per year*)

Age	Class (transformed)				Class (untransformed)			
(years)	Fac.[2]	Off.	Ret.	War.	Fac.	Off.	Ret.	War.
1	2.20	2.39	1.87	2.05	1.32	1.86	1.80	2.33
5	2.29	2.44	1.92	2.14	1.34	1.90	1.75	2.32
10	2.42	2.51	2.00	2.27	1.36	2.06	1.68	2.31
20	2.73	2.64	2.17	2.59	1.38	3.01	1.50	2.26
30	3.11	2.73	2.34	3.03	1.38	6.17	1.27	2.18
40	3.58	2.72	2.51	3.67	1.35	31.80	1.02	2.14
50	4.15	2.54	2.63	4.71	1.27	—	0.77	2.21

[1] Sources: Hulten and Wykoff, (1976, 1977).
[2] Fac.=Factory; Off.=Office; Ret.=Retail; War.=Warehouse.

culated the average annual dollar decline with these rates and found that these amounts fell with age. Thus, while five of these patterns are less accelerated than geometric, they are more accelerated than straight-line. The untransformed factory pattern is nearly geometric, and the untransformed retail store and warehouse patterns are more accelerated than geometric. The rates of depreciation range roughly between 1.3 percent and 2.3 percent for untransformed and between 2 percent and 3 percent for transformed data.

We conclude that depreciation of structures is accelerated vis-à-vis straight-line, and we statistically reject "one-hoss-shay" decay rather decisively. The rates of depreciation our study has produced are all considerably lower than those implied by published guideline lives.[11] Our rates fall in the 1.5 percent to 3 percent range while the guideline lives imply 5 percent to 7 percent rates of depreciation.

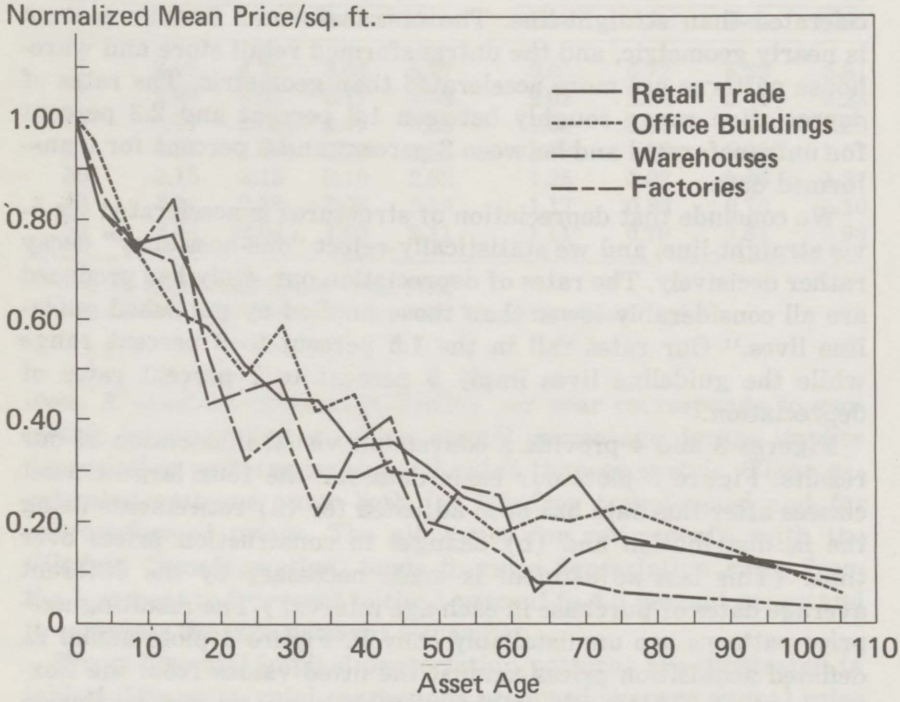
Figures 3 and 4 provide a convenient visual illustration of our results. Figure 3 plots our basic data for the four largest asset classes after this data has been adjusted for (a) retirements using the L_0 distribution and (b) changes in construction prices over time. (This last adjustment is made necessary by the different average dates of purchase in each age interval.) The resulting age-price patterns are unmistakably convex. Figure 4 plots actual L_0 deflated acquisition prices against the fitted values from the Box-Cox analysis. The result, again, strongly supports our conclusion that economic depreciation is accelerated relative to a straight-line pattern. Our evidence thus suggests rather strongly that the conventional wisdom—that buildings depreciate like a "one-hoss shay"—is unwarranted. The implication for tax policy is that accelerated depreciation ought to be allowed for structures, but that the currently allowed implicit guideline rates are probably too large. (We are assuming here that economic depreciation is the desired norm for tax depreciation allowances.)

Cross-Sectional Results for Office Buildings

The results described in the preceding section were obtained by pooling used-asset price data for a number of years. That is, a 5-year-old office building in 1948 was pooled with a 20-year-old office building purchased in 1965. Thus, while the pooled analysis

[11] U.S. Department of the Treasury, Internal Revenue Service, *Depreciation Guidelines and Rules, Revenue Procedure 62–21*, Revised 1964.

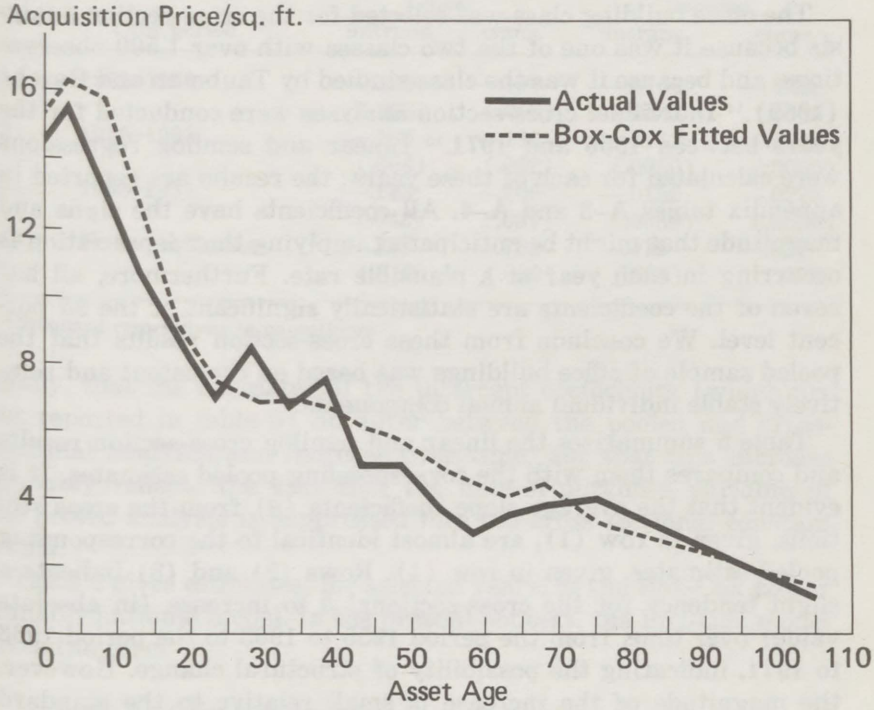
FIGURE 3.—*Mean asset price by age interval trans/defl.*



provides a reasonably good statistical fit to the underlying data and indicates a convex age-price pattern, as is seen in figures 2 and 4, the possibility that pooling introduces a systematic bias into the results does exist. The argument that such a bias might exist was advanced by Taubman (1976), who argued that there have been significant changes over the postwar years in the business tax code (as it relates to tax depreciation) as well as significant variations in the rate of inflation.[12] Two identical buildings might have different values in different years—and thus different observed purchase prices—simply because the present value of the net income stream has been altered by changes in the tax code or the inflation rate. Thus, if there is a correlation between the age of buildings in our sample and the years in which the purchases

[12] Major changes in the depreciation rules occurred in 1954, 1962, and 1971; numerous other changes have also occurred. Over the period 1955–1971, the average annual growth of the Boechk construction price index was 3.86 percent, for the period 1958–1971, the average annual change of the Bureau of Census Composite Index was 2.93 percent.

FIGURE 4.—*Office building: fitted vs. actual price by age interval trans.*



occurred, the observed age-price patterns of figures 1 and 3 could be biased upward or downward.

A positive correlation between asset age and date of purchase is contained in our sample; for office buildings, the correlation coefficient is 0.13. Furthermore, the estimated Box-Cox parameters for the office building class imply that the regressions are cubic in date, with very small and statistically insignificant coefficients. The possibility of a misspecification associated with the date-of-purchase variable does therefore exist. As noted earlier, we have attempted to correct for any bias due to pooling by breaking the pooled sample into subsamples corresponding to individual years, and then analyzing each year separately. The effects of time on used-asset prices [13] are naturally held constant by this method, and

[13] The use of annual cross sections does not, of course, remove all effects associated with the passage of time. Improvements in the quality of buildings will introduce a time dimension into the data for any year. To see this in its simplest form, assume that depreciation, inflation, and quality change are

if the results are consistent with those obtained from the pooled analysis, then we can conclude that biases in the latter due to differing dates of purchase are small.

The office building class was selected for the cross-section analysis because it was one of the two classes with over 1,500 observations, and because it was the class studied by Taubman and Rasche (1969).[14] Individual cross-section analyses were conducted for the years between 1955 and 1971.[15] Linear and semilog regressions were calculated for each of these years; the results are reported in appendix tables A–3 and A–4. All coefficients have the signs and magnitude that might be anticipated, implying that depreciation is occurring in each year at a plausible rate. Furthermore, all but seven of the coefficients are statistically significant at the 95 percent level. We conclude from these cross-section results that the pooled sample of office buildings was based on consistent and relatively stable individual annual components.

Table 5 summarizes the linear and semilog cross-section results and compares them with the corresponding pooled estimates. It is evident that the average slope coefficients (β) from the cross sections, given in row (1), are almost identical to the corresponding pooled estimates, given in row (4). Rows (2) and (3) indicate a slight tendency for the cross-sectional $\hat{\beta}$ to increase (in absolute value) over time, from the period 1955 to 1963 to the period 1963 to 1971, indicating the possibility of structural change. However, the magnitude of the increase is small relative to the standard errors, so it is difficult to discern a significant effect.[16] We note,

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occurring at constant, exponential rates β, γ, and δ respectively. Suppose, further, that the price of an S-year-old asset at time t is given by

$$Y(t,S) = \alpha\, e^{-\beta S}\, e^{\gamma t}\, e^{\delta(t-S)} \qquad \text{(A)}$$

The term $\delta(t\text{-}S)$ allows for differences in the quality of different vintages. Equation (A) can be written

$$Y(t,S) = \alpha e^{-(\beta+\delta)S}\, e^{(\gamma+\delta)t}, \qquad \text{(B)}$$

implying that β, γ, and δ cannot be identified using data on asset price, $Y(t,S)$. This result is due to Hall (1968), and it should be kept in mind when interpreting the results of this paper.

If quality change varies over time, the appropriate analog to (A) will imply cross sections that depend on t. This effect can also arise if past changes in the tax code are embodied in the design of assets.

[14] Thus we facilitate comparison of our results with those derived by Taubman and Rasche—see the final section of this paper.

[15] These years were selected because they contain a reasonably large number of observations.

[16] In view of footnote 13, the stability of the cross-sectional estimates indicates that, to the extent that embodied quality changes are important, it is a smooth process.

TABLE 5.—*Linear and semilog cross-section results*

	Time period	Average estimated slope coefficients Linear untrans.	Linear trans.	Semilog untrans.	Semilog trans.
(1)	1955–1971	—.139	—.198	—.0146	—.0258
		(.053)	(.050)	(.0043)	(.0043)
(2)	1955–1963	—.136	—.190	—.0142	—.0245
		(.064)	(.062)	(.0050)	(.0050)
(3)	1963–1971	—.143	—.206	—.0156	—.0276
		(.042)	(.037)	(.0036)	(.0036)
(4)	Pooled estimates	—.127	—.182	—.015	—.027
		(.010)	(.009)	(.001)	(.002)

[1] Standard errors given in parentheses.

finally, that the estimates of the intercept coefficients (which are not reported in table 5) do differ between the pooled and cross-sectional analyses and increase over time, but that this increase probably reflects the fact that the date-of-purchase variable of the pooled analysis is suppressed into the cross-sectional constant terms.

Table 6 gives estimates, for selected years, of the Box-Cox power transformational model. In the present context, the Box-Cox model takes the form

TABLE 6.—*Office buildings: Box-Cox grid search of selected years*

(a) Year	(b) Box-Cox estimates of (θ_1, θ_2)	(c) Box-Cox adjusted log-likelihoods	(d) Adjusted log-likelihoods
TRANSFORMED DATA (θ_1=.212, θ_2=.714)			
1956	(0.45, 0.40)	—144.57	—146.94
1959	(0.20, 0.80)	—171.25	—171.29
1962	(0.20, 1.20)	—211.55	—211.62
1965	(0.35, 0.80)	—349.16	—350.89
1968	(0.30, 0.80)	—333.58	—334.32
1971	(0.30, 0.80)	—221.34	—221.85
UNTRANSFORMED DATA (θ_1=.247, θ_2=.384)			
1956	(0.55, 0.20)	—149.35	—151.84
1959	(0.25, 0.40)	—178.81	—178.81
1962	(0.20, 0.60)	—221.32	—221.43
1965	(0.40, 0.00)	—365.14	—367.98
1968	(0.40, 0.60)	—350.02	—352.95
1971	(0.40, 0.60)	—238.41	—240.22

$$\frac{Y^{\theta_1}-1}{\theta_1}=\alpha+\beta\,\frac{S^{\theta_2}-1}{\theta_2}+\varepsilon, \qquad (1)$$

where Y denotes acquisition price per square foot and S, the age of the building when purchased, θ_1 and θ_2 are unknown form parameters, and α and β are the respective unknown intercept and slope parameter. When $(\theta_1,\theta_2)=(1,1)$, equation (1) is a linear regression model. When $\theta_1 \rightarrow 0$, the left-hand side of equation (1) becomes ln Y, so that $(\theta_1,\theta_2)=(0,1)$ implies that equation (1) is a semilog regression. Other restrictions on (θ_1,θ_2) are possible: for example, $(\theta_1,\theta_2)=(1,3)$ yields a cubic regression.

The parameters of equation (1) were estimated using a grid search method.[17] The resulting estimates of the θ's are given in column (b) of table 6. Because of the costs involved, the estimates were calculated only for every third year of the period 1955 to 1971. The estimates of (θ_1,θ_2) show a high degree of stability and, more importantly, are quite close to the pooled estimates of (θ_1,θ_2): (.247,.384) for the untransformed data and (.212,.714) for the transformed. Intuitively, this means that the age-price patterns are relatively stationary over time.

This last statement can be made statistically precise by considering the adjusted log-likelihood function associated with equation (1). Assuming that the error term ε is independently normally distributed, the log-likelihood function is given by

$$L=(\theta_1-1)\sum_{i=1}^{N}\ln Y_i-\frac{N}{2}(\ln 2\pi+\ln\hat{\sigma}^2+1), \qquad (2)$$

where $\hat{\sigma}$ is the estimated standard error of the regression and N is the number of observations. The normality of ε implies the approximate normality of L for large N. Furthermore, the statistic $\lambda=-2(L(\theta_1^*,\theta_2^*)-L)$ is approximately chi-square with two degrees of freedom when $(\theta_1,\theta_2)=(\theta_1^*,\theta_2^*)$. (Note that $L(\theta_1^*,\theta_2^*)$ is the value of equation (1) calculated under the assumption that $\theta_1=\theta_1^*$ and $\theta_2=\theta_2^*$.) Column (c) of table 6 gives values for L; column (d) gives values for $L(.212,.714)$ and $L(.247,.384)$—the values of the adjusted log-likelihood function calculated under the assumption that the pooled estimates of (θ_1,θ_2) are correct. The statistic λ is calculated by multiplying the difference between columns (c) and (d) by two, and comparing the result with the 5 percent critical value of a chi-square variate with two degrees of freedom: 5.99. So calculated, λ is less than 5.99 in all years. Thus,

[17] The parameter θ_1 was searched in increments of 0.05, and θ_2 in increments of 0.2.

we cannot reject the hypothesis that the pooled estimates and the cross-section estimates of the pair (θ_1,θ_2) are the same.

This statistical result is of considerable importance. Because the cross-sectional and pooled estimates of (θ_1,θ_2) are not significantly different under the λ test,[18] and because figures 2 and 4 clearly indicate a convex age-price pattern for the pooled estimates, we conclude that yearly cross-sectional estimates are consistent with the convex pattern. This conclusion implies in turn that the convexity of the age-price pattern is not a result of the misspecification of time in the pooled analysis.

The conclusion that the age-price pattern is convex can be further assessed by considering the form of the Box-Cox function [equation (1)] when $\varepsilon=0$. The first derivative is given by

$$\frac{dY}{dS}=\beta\, Y^{1-\theta_1}S^{\theta_2-1}. \quad (3)$$

Since Y and S are nonnegative, the slope of the age-price pattern will be nonpositive if, and only if, $\beta<0$. All of our estimates of β are, in fact, less than 1. This result implies that acquisition-price per square foot declines with age (Y,S greater than zero). The second derivative of equation (1) is given by

$$\frac{d^2Y}{dS^2}=\frac{\theta_2-1}{S}\frac{dY}{dS}+\frac{1-\theta_1}{Y}\left(\frac{dY}{dS}\right)^2. \quad (4)$$

Note that when the regression is linear, $(\theta_1,\theta_2)=(1,1)$, equation (3) is constant and equation (4) equals zero; when the regression is semilog, the first derivative equals β and the second derivative equals β^2/Y. $\beta<0$ implies that the second derivative is everywhere greater than zero, and the age-price pattern is strictly convex whenever $\theta_1\leqq 1$ and $\theta_2\leqq 1$.

Because of the traditional importance of the "one-hoss-shay" hypothesis, we pursue the issue of nonconvexity one step further and consider the implication of $\theta_2>1$. If $\hat{\beta}$ is small and uniformly negative, $\theta_1>1$ and $\theta_2>1$ imply a concave age-price pattern. However $\theta_1>1$ can be strongly rejected in every year considered in table 7. Note that the second term on the right-hand side of equation (4) will eventually come to dominate d^2Y/dS^2 as age S increases. Thus, since this term is positive for $\theta_1<1$, the age-price pattern must ultimately become convex as age increases. If θ_2 is positive and sufficiently large, however, then the first term on the right-hand side of equation (4) will be negative and will dominate, at least in the early years. In this case, the early years of asset life

[18] The term "λ test" will be used to signify "asymptotic likelihood ratio test."

will be characterized by a concave age-price pattern if β is small. The overall pattern would be shaped like a backward S. While a backward S is not consistent with the pure "one-hoss-shay" hypothesis, it is consistent with a variant of the hypothesis in which (a) buildings are "one-hoss-shay" in the early years of their life, and (b) these buildings are retired at different points of time.

The preceding discussion suggests a direct test of convexity based on $(\hat{\beta},\hat{\theta}_1,\hat{\theta}_2)$. Note that the estimates of $(\theta_1.\theta_2)$ in column (b) of table 6 are, with one exception, less than one. Although not reported, it is also the case that $\hat{\beta}$ is always negative. Thus, the Box-Cox maximum likelihood point estimates consistently imply a convex age-price pattern. However, the adjusted likelihood functions are relatively flat over different values of θ_2 (but not θ_1). When the λ test discussed earlier is applied to the hypothesis that $\theta_2>1$, this hypothesis is rejected only in the 1965 and 1968 cross sections. These two years are the ones with the largest number of observations, and it is possible that more data points in the other years would lead to more precise estimates and thus to the rejection of the alternative hypothesis $\theta_2>1$ (and therefore to the acceptance of a strictly convex age-price pattern). The data is, unfortunately, not available, and the possibility that the age-price pattern is concave over some asset ages must be considered.

We emphasize, however, that the cross-sectional maximum likelihood point estimates do not indicate the backward-S-shaped age-price pattern (except in 1962 with the transformed data). On the contrary, these estimates imply a strictly convex pattern in most years. The point of this discussion is to note that the backward-S-shaped pattern cannot be statistically ruled out in the years 1956, 1959, 1962, and 1971.

We observe, finally, that the hypotheses of linear, geometric, and cubic age-price patterns can be rejected using the λ test in every year except 1962 (where geometric cannot be rejected). The adjusted log-likelihoods are given in tables 7 and 8. Note also that the geometric (semilog) likelihoods generally outperform the linear and cubic cases, although the λ test is inapplicable because the three cases are not nested. The adjusted log-likelihood associated with a third degree polynomial in age is also given. The polynomial likelihoods cannot be compared with either the Box-Cox or semilog functions, since, again, they cannot be derived from restrictions on the relevant parameters. We note, however, that the adjusted log-likelihoods are uniformly larger under Box-Cox and generally larger under the semilog regressions. The linear and cubic functions can be derived from the polynomial functions (using the

TABLE 7.—*Office buildings (untransformed): adjusted log-likelihoods*

Year	Semilog $\ln Y=\alpha+\beta S+\varepsilon$ $(\theta_1=0)$	Linear $Y=\alpha+\beta S+\epsilon$ $(\theta_1=1)$	3rd degree polynomial $Y=\alpha+\beta_1 S+\beta_2 S^2+\beta_3 S^3+\varepsilon$ $(\theta=1)$	Cubic $Y=\alpha+\beta S^3+\epsilon$ $(\theta_1=1)$
1955	—135.7	—163.0	—162.8	—163.3
1956	—159.2	—154.9	—153.4	—158.4
1957	—156.9	—167.7	—166.6	—170.2
1958	—182.2	—210.4	—210.3	—210.7
1959	—182.4	—195.3	—194.9	—197.1
1960	—249.7	—257.2	—256.9	—257.9
1961	—217.9	—235.7	—235.7	—235.9
1962	—222.7	—238.9	—238.7	—238.8
1963	—322.3	—339.2	—338.0	—340.7
1964	—288.4	—298.6	—298.3	—299.7
1965	—382.7	—386.8	—384.3	—391.8
1966	—300.1	—318.6	—317.5	—321.1
1967	—302.7	—336.8	—332.4	—337.3
1968	—358.8	—363.5	—361.4	—373.1
1969	—275.9	—284.8	—283.4	—287.5
1970	—220.0	—216.9	—213.5	—226.6
1971	—245.2	—248.5	—247.6	—252.8

TABLE 8.—*Office buildings (transformed): adjusted log-likelihoods*

Year	Semilog $\ln Y=\alpha+\beta S+\epsilon$ $(\theta_1=0)$	Linear $Y=\alpha+\beta S+\epsilon$ $(\theta_1=1)$	3rd degree polynomial $Y=\alpha+\beta_1 S+\beta_2 S^2+\beta_3 S^3+\varepsilon$ $(\theta=1)$	Cubic $Y=\alpha+\beta S^3+\epsilon$ $(\theta_1=1)$
1955	—131.96	—162.90	—162.74	—163.32
1956	—153.27	—153.32	—151.78	—158.48
1957	—150.19	—166.95	—165.91	—170.82
1958	—176.99	—209.75	—209.68	—210.50
1959	—174.12	—191.48	—191.14	—195.08
1960	—242.51	—255.02	—218.07	—257.06
1961	—210.58	—232.07	—232.07	—232.81
1962	—213.37	—231.82	—231.67	—232.42
1963	—313.93	—337.20	—336.79	—340.34
1964	—278.74	—294.91	—294.61	—297.53
1965	—362.64	—380.23	—378.21	—391.41
1966	—284.98	—316.36	—314.69	—321.85
1967	—289.31	—301.96	—297.01	—308.96
1968	—340.22	—358.52	—355.34	—374.00
1969	—260.90	—276.02	—274.05	—282.16
1970	—198.20	—209.27	—203.47	—225.47
1971	—226.28	—242.66	—240.69	—251.82

restrictions $\beta_2=\beta_3=0$ and $\beta_1=\beta_2=0$, respectively) ; surprisingly, the corresponding λ test rejects linear vis-à-vis polynomial in only three years. The cubic form is rejected in a majority of the cross sections and is uniformly rejected after 1965.

Summary and Comparison with Other Studies

Our basic findings can be stated briefly: office buildings appear to depreciate at a rate faster than straight-line depreciation, and this result holds for both pooled data and for data analyzed as a sequence of annual cross sections. The treatment of time (and thus inflation, etc.) does not, therefore, appear to bias the pooled office building results (to any great extent) on the convexity of the age-price pattern.

These results differ sharply from those reported by Taubman and Rasche (1969), who studied a sample of average rental income, operating costs, and vacancy rates for 600 office buildings between the years 1951 and 1965. Their data distinguish four age categories: less than 10 years old, 10 to 25 years old, 25 to 40 years old, and over 40 years old. Taubman and Rasche derived an average annual rental series (per square foot) for each age S by an elaborate interpolation procedure and then constructed an average asset price series, Y, by assuming that asset price equals the present value of the rental income net of operating costs. The asset price series constructed in this way yielded a highly concave age-price pattern. Rates of depreciation—$(1/Y)$ (dY/dS) in our notation—are continuously increasing and do not exceed 1.00 percent until $S>25$ in their 1951 cross section, or until $S>39$ in their 1960 cross section.[19]

We have fitted the Taubman and Rasche results to the Box-Cox model in order to determine the implied values of (θ_1,θ_2).[20] Using the same grid search method applied to our samples, we find that in both 1951 and 1960 the point estimates of (θ_1,θ_2) are (1.10,2.40). These values, when imposed on our data, are strongly rejected using the λ criterion. It is worth noting, however, that Taubman and Rasche have as a maintained hypothesis the assumption that asset value is equal to the present value of the rental income flow. This assumption, which is not necessary to our statistical estimation of

[19] See Taubman and Rasche (1969), Table IIa.

[20] Data from Taubman and Rasche's (1969) Table IIa was, again, used for this purpose. We used their estimated present values, which are based on a discount rate of 10 percent.

economic depreciation,[21] implies a competitive market equilibrium with perfect foresight. Furthermore, Taubman and Rasche have average annual data for only four broad age categories and therefore only four points on the age-price curve. Their data consists of grossly aggregated averages provided them by a real estate association. Our data, on the other hand, is derived from a stratified sample based on the Internal Revenue Service's taxpayers compliance file and has numerous points on the age-price curve.[22] It is, of course, true that we do not know the reasons why specific buildings in our sample were resold, and, therefore, we cannot be sure that they are representative of the entire population of office buildings. Furthermore, we note again that we used the raw data unweighted by sampling probabilities, for reasons discussed elsewhere (Hulten and Wykoff, 1977). We do not, however, have any particular reason to believe that systematic biases are present in our sample.[23]

The studies reported by Coen (1975; 1976) provide an alternative approach to the measurement of depreciation of nonresidential structures. The neoclassical theory of investment postulates that investment depends, in part, on the rental cost of capital. The rental, in turn, depends on physical depreciation. Coen models the rental price using different assumptions about depreciation—sum-of-years-digits, geometric, straight-line, and "one-hoss-shay"—and fits a neoclassical investment model using each accounting depreciation variant. The variant yielding the lowest standard error (and plausible values for the estimated parameters) was selected as the most probable physical depreciation pattern. In the 21 manufacturing industries studied by Coen (1975), "one-hoss-shay" was found to be superior for structures in 11 industries, straight-line in 5, and geometric and sum-of-years-digits in 5. However, in

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[21] We do use this assumption (Hulten and Wykoff, 1975) to calculate the relative physical efficiency of the various types of buildings in our study. We do not, therefore, wish to argue that the assumption is wrong, but only to point out that it is an additional hypothesis not required for the direct measurement of economic depreciation from our data.

[22] See U.S. Department of the Treasury, Office of Industrial Economics (1975) for a description of the sampling methods underlying the OIE real property study.

[23] There may be, for example, a systematic bias resulting from the location of older buildings nearer to the centers of the various cities in the OIE sample. It might also be the case that buildings appear on the market at a discount because they no longer meet the business needs of the sellers. However, while these (and other) possibilities exist, it seems more probable that most buildings are sold for reasons related to the general business and financial circumstances of the owners and not to the characteristics of the structures themselves.

a subsequent paper Coen (1976) modified these results significantly. "One-hoss-shay" was accepted in only 2 industries, and sum-of-years-digits and geometric in 14. The second set of results is quite consistent with the results of our work. Although manufacturing structures are primarily factories, and the analysis is based on the form of physical depreciation, denoted by the decline in relative efficiency or by the mortality sequence, we observe that a near geometric form of physical depreciation implies a near geometric form of economic depreciation and interpret Coen's results as consistent with our own.

The results of this paper and those of Coen (1976) run counter to the intuitive argument made by some people that buildings are refurbished and maintained indefinitely at or near their original productive capacity. But could the intuitive argument not also be applied to automobiles, machine tools, and other equipment? Theoretically, these assets could also be maintained and repaired so that they would provide the same productive services as new equipment. Yet studies tend to indicate that equipment also has a convex age-price profile.[24] It may be that intuition has been a misleading guide in understanding economic depreciation.

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[24] See, for example, Beidleman (1976) and Wykoff (1970).

TABLE A-1.—*Pooled Box-Cox point estimates: office buildings* *

	θ_1	θ_2	θ_3	α	β	γ	Log L (R^2)
UNTRANSFORMED							
Linear	1	1	1	3.63	—.127	.177	—6021.1
				**	(.010)	(.020)	(.123)
Geometric	0	1	1	1.104	—.015	.022	—5583.9
				**	(.001)	(.002)	(.203)
Box-Cox							
2 Constraints	.277	.277	.277	.105	—.207	.480	—5515.2
	(.011)	(.011)	(.011)	(.321)	(.012)	(.043)	
1 Constraint	.248	.610	.610	.603	—.083	.153	—5503.5
	(.011)	(.092)	(.092)	(.339)	(.022)	(.056)	
0 Constraints	.247	.384	3.289	2.452	—.158	.379E–5	—5481.49
	(.011)	(.098)	(.659)	(.178)	(.039)	(.013E–4)	
TRANSFORMED							
Linear	1	1	1	4.517	—.182	.162	—5893.3
				**	(.009)	(.018)	(.220)
Geometric	0	1	1	1.110	—.027	.022	—5302.6
				**	(.001)	(.002)	(.403)
Box-Cox							
2 Constraints	.294	.294	.294	.231	—.317	.467	—5282.7
	(.011)	(.011)	(.011)	(.319)	(.011)	(.041)	
1 Constraint	.209	.801	.801	1.010	—.071	.069	—5236.2
	(.011)	(.059)	(.059)	(.208)	(.013)	(.017)	
0 Constraints	.212	.714	3.154	2.34	—.094	.6E–5	—5220.4
	(.011)	(.060)	(.634)	(.165)	(.017)	(.16E–4)	

* Coefficients derived from the equation $\frac{Y^{\theta_1}-1}{\theta_1}=\alpha+\beta\frac{S^{\theta_2}-1}{\theta_2}+\gamma\frac{t^{\theta_3}-1}{\theta_3}$.

** A computer program was used which did not provide standard errors of constant terms. Standard errors for other coefficients are given in parentheses.

TABLE A-2.—*Pooled polynomial point estimates* [1]

	α [2]	β_1	β_2	β_3	γ_1	γ_2	δ
Untransformed estimates	12.81	—.271 (5.4)	.0026 (2.3)	—.000001 (—1.2)	.270 (6.9)	.002 (1.7)	.000011 (0.85)
Transformed estimates	12.19	—.343 (—7.8)	.0031 (3.1)	.000009 (1.7)	.302 (8.8)	.0016 (1.6)	.000009 (0.78)

[1] t statistics given in parentheses.

[2] Estimates derived from the equation.

$$Y=\alpha+\beta_1 S+\beta_2\cdot S^3+\gamma_1 t+\gamma_2 t^2+\delta I,$$

where I is a variable denoting regional income t.

TABLE A–3.—*Office buildings (transformed): annual regressions* [1]

Year	$Y=\alpha+\beta S+\epsilon$ [2]			$\ln Y=\alpha+\beta S+\epsilon$		
	$\hat{\alpha}$	$\hat{\beta}$	R^2	$\hat{\alpha}$	$\hat{\beta}$	R^2
1955	14.18	—0.207	0.07	2.20	—0.018	0.13
	(5.29)	(—1.709)		(12.97)	(—2.391)	
1956	13.54	—0.214	0.36	2.48	—0.031	0.42
	(13.62)	(—5.176)		(19.26)	(—5.822)	
1957	12.70	—0.188	0.25	2.34	—0.025	0.38
	(10.58)	(—3.984)		(19.74)	(—5.388)	
1958	17.14	—0.245	0.12	2.65	—0.027	0.38
	(8.10)	(—2.695)		(23.40)	(—5.556)	
1959	13.21	—0.152	0.20	2.50	—0.026	0.49
	(12.04)	(—3.669)		(26.28)	(—7.212)	
1960	14.75	—0.168	0.13	2.41	—0.021	0.20
	(11.40)	(—3.151)		(19.36)	(—4.075)	
1961	14.90	—0.169	0.11	2.47	—0.019	0.21
	(9.87)	(—2.736)		(20.85)	(—3.966)	
1962	15.36	—0.177	0.14	2.58	—0.024	0.30
	(10.40)	(—3.236)		(20.34)	(—5.133)	
1963	14.52	—0.187	0.15	2.50	—0.030	0.37
	(13.92)	(—4.103)		(26.60)	(—7.408)	
1964	15.75	—0.195	0.17	2.57	—0.023	0.27
	(13.19)	(—4.060)		(24.75)	(—5.474)	
1965	14.58	—0.161	0.22	2.44	—0.022	0.29
	(14.95)	(—5.392)		(22.38)	(—6.523)	
1966	14.56	—0.147	0.17	2.52	—0.023	0.43
	(13.12)	(—4.223)		(27.74)	(—8.068)	
1967	16.73	—0.184	0.21	2.76	—0.028	0.44
	(15.18)	(—4.705)		(28.77)	(—8.137)	
1968	19.14	—0.266	0.40	2.89	—0.031	0.56
	(19.72)	(—8.187)		(35.61)	(11.409)	
1969	18.52	—0.242	0.28	2.78	—0.027	0.38
	(13.47)	(—5.443)		(22.46)	(—6.750)	
1970	17.56	—0.237	0.57	2.93	—0.035	0.58
	(16.57)	(—9.35)		(19.33)	(—9.550)	
1971	18.04	—0.232	0.38	2.88	—0.029	0.48
	(13.74)	(—6.609)		(21.07)	(—8.05)	

[1] *t* statistics given in parentheses.

[2] *Y* denotes acquisition price per square foot; *S* denotes the age of the building when purchased.

TABLE A–4.—*Office buildings (untransformed): annual regressions* [1]

Year	$Y=\alpha+\beta S+\epsilon$ [2]			No.obs.	$\ln Y=\alpha+\beta S+\epsilon$		
	$\hat{\alpha}$	$\hat{\beta}$	R^2		$\hat{\alpha}$	$\hat{\beta}$	R^2
1955	14.14 (5.26)	—0.161 (—1.32)	0.04	40	2.18 (12.87)	—0.008 (—1.09)	0.03
1956	13.50 (13.14)	—0.171 (—4.01)	0.25	49	2.46 (18.92)	—0.022 (—4.00)	0.25
1957	12.65 (10.39)	—0.145 (—3.02)	0.16	50	2.32 (19.38)	—0.015 (—3.12)	0.17
1958	17.14 (8.00)	—0.191 (—2.07)	0.08	53	2.63 (23.41)	—0.017 (—3.58)	0.20
1959	13.19 (11.24)	—0.102 (—2.30)	0.09	57	2.45 (25.34)	—0.014 (—3.70)	0.20
1960	14.71 (11.03)	—0.111 (—2.01)	0.06	70	2.38 (19.09)	—0.010 (—1.91)	0.05
1961	14.81 (9.26)	—0.091 (—1.38)	0.03	63	2.46 (20.75)	—0.009 (—1.94)	0.06
1962	15.37 (9.32)	—0.106 (—1.73)	0.05	64	2.55 (20.12)	—0.014 (—2.93)	0.12
1963	14.55 (13.67)	—0.148 (—3.18)	0.10	94	2.47 (26.43)	—0.019 (—4.71)	0.19
1964	15.64 (12.52)	—0.121 (—2.42)	0.07	82	2.54 (24.57)	—0.012 (—2.91)	0.10
1965	14.50 (13.98)	—0.102 (—3.22)	0.09	108	2.39 (21.66)	—0.010 (—2.89)	0.07
1966	14.62 (12.85)	—0.100 (—2.80)	0.08	88	2.45 (26.74)	—0.009 (—2.97)	0.09
1967	16.13 (9.76)	—0.061 (—1.05)	0.01	86	2.71 (28.06)	—0.016 (—4.73)	0.21
1968	19.15 (18.80)	—0.212 (—6.22)	0.27	104	2.84 (34.54)	—0.019 (—7.07)	0.33
1969	18.42 (11.95)	—0.169 (—3.40)	0.13	77	2.72 (21.73)	—0.016 (—3.89)	0.17
1970	17.79 (15.02)	—0.196 (—6.93)	0.42	68	2.83 (18.45)	—0.022 (—6.06)	0.36
1971	18.20 (12.78)	—0.178 (—4.68)	0.24	72	2.78 (20.25)	—0.017 (—4.60)	0.23

[1] *t* statistics given in parentheses.

[2] *Y* denotes acquisition price per square foot; *S* denotes the age of the building when purchased.

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COMMENT

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If our income tax is to be just that—an income tax—tax depreciation allowances must approximate as closely as possible actual changes in the value of capital assets over the accounting period. To the extent that tax allowances fall short of actual depreciation, the "income tax" becomes a tax on capital itself; and to the extent that tax allowances exceed actual depreciation, the "income tax" becomes a subsidy to capital. Nearly all tax scholars and practitioners would agree with these propositions and would use them to defend proposals for reforming tax depreciation policy. But this does not mean that all would arrive at the same reform proposals. Some will argue that liberalization of tax allowances has gone too far, that the current income tax subsidizes capital, and that reform should entail increases in service lives permitted for tax purposes and/or slower write-off methods over these service lives. Others hold precisely the opposite view. Still others admit that tax allowances might be liberal but urge us to keep them that way (or even liberalize them further) in order to provide incentives for capital formation.

These divergent views on depreciation reform arise not so much from differences in conceptual frameworks as from differences in perceptions of reality. Are tax allowances currently in excess of actual depreciation or are they not? Hulten and Wykoff show that it is possible to attack this question in a scientific way, and they present us with empirical applications of their own method. Before noting my doubts about their approach and the problems I believe they have neglected, let me say that the question addressed is a very difficult one for which we need every bit of evidence we can get, that no method of estimating actual depreciation is beyond reproach, and that applications and comparisons of various methods ought to be enthusiastically encouraged.

Hulten and Wykoff estimate depreciation rates of nonresidential structures by studying the relationships between market prices of new and used buildings. The data employed were collected by the Office of Industrial Economics of the Internal Revenue Service. Each observation in the sample gives information on a building that was traded—the price paid for it, its age at the time of the trade, and the date of the trade. Contrary to a view I had expressed earlier, evidently there is enough trading of some capital goods to generate samples of data that are large enough to permit statistical analyses; I hope that more surveys of this type will be undertaken in the future.

Hulten and Wykoff begin by positing that the value of a building is a function of its age and its date of acquisition. The notion that the age of a building may affect its value is easily understood. Standard capital theory tells us that the value of an asset should equal the sum of the discounted net revenues the asset will generate in the future (net of operating costs but not depreciation). Aging will diminish this value even if net revenue is constant in each year of an asset's life, because the stream of future net revenues grows shorter. In addition, the asset may lose efficiency or become obsolescent as it ages, in which case net revenue will decline with age and the value of the asset will fall more rapidly. Thus, if we consider two buildings traded in 1970, one 10 years old and the other 20 years old, we should expect that the first would command a higher price than the second.

The influence of acquisition date on the value of an asset is less apparent. Again consider two buildings, both 10 years old, one traded in 1960 and the other in 1970. Why should their purchase prices have differed? Perhaps the most important reason is that the general level of prices may have changed between 1960 and 1970, with building prices undergoing the same inflation or deflation as those of other commodities. If inflation has been occurring, then we should expect that a 10-year-old building would sell for more in 1970 than in 1960; that is, holding age constant, we should expect the purchase price of a building to be positively related to its date of purchase. Note, however, that the acquisition date is serving here as a proxy for inflation and that the estimated effects of acquisition date will be determined largely by changes in the general price level during the particular sample period being studied.

The Hulten-Wykoff hypothesis can be capsulized by writing $Y(S,t)$ as the value of a building of age S in year t. Assuming that the purchase price of a traded building equals its value, the OIE data provide observations from which the general contours of the relationship between Y and S and t can be estimated. The authors test their hypothesis using flexible functional forms in S and t, such as Box-Cox power transformations and polynomial expressions. The detailed results reported for office buildings confirm the expected negative influence of S on Y, with t held constant;

but the influence of t is not so firmly established. In the polynomial forms, t has a positive and statistically significant effect on Y, S held constant; in the Box-Cox forms (unconstrained), the influence of t is again positive but quite insignificant. In view of the inflation that occurred during the sample period (postwar years through 1971), the Box-Cox results are certainly puzzling in this respect.

The next step in the analysis is to calculate depreciation rates from the estimated equations relating value to age and acquisition date. The results, summarized in tables 3 and 4 of the paper, generally indicate that depreciation of buildings is more accelerated than straight-line (in some cases even more accelerated than declining-balance), but that the depreciation rates are considerably lower than those consistent with current tax service lives (i.e., 1962 guideline lives). The policy implications are, therefore, that accelerated depreciation of buildings ought to be maintained but that tax service lives ought to be lengthened. Since the little evidence we have on actual depreciation practices suggests that buildings are being depreciated at lives shorter than the guideline lives, the increase in *effective* tax lives needed to equate tax and actual depreciation may be much greater than the increase in nominal, or statutory, tax lives. For example, Ture's (1967) analyses of Treasury survey data indicated that manufacturers were depreciating their structures over a 23-year average life compared with guideline lives of 45 to 50 years. Regarding the basic empirical issue noted at the beginning of my comments, the evidence presented by Hulten and Wykoff suggests that tax depreciation allowances have probably been outstripping actual depreciation, although the authors do not provide us in this paper with overall annual estimates of these two measures of depreciation.

The authors do not clearly explain the procedures they followed in calculating depreciation rates. It is quite important that readers understand the nature of these calculations and the interpretation to be given the rates reported. Using their empirical estimates of the $Y(S,t)$ relationship, they arrive at their rates by holding t (vintage or acquisition date) constant and computing the percentage change in Y as S increases. Thus, the first-year depreciation rate on vintage 1 buildings is $[Y(0,1) - Y(1,1)]/Y(0,1)$, the second-year depreciation rate is $[Y(1,1) - Y(2,1)]/Y(1,1)$, etc. The rates thus derived are specific to a particular vintage of buildings. As in their previous work, Hulten and Wykoff select 1970 as the acquisition date for calculating depreciation rates. Had they selected 1960, 1977, or 1980, the rates would generally not have been the same. Given the form and coefficients of the polynomial version of the office-buildings equations in the appendix table A–1, the calculated depreciation rates will be uniformly lower the later the acquisition date. (This is much less true of the estimated Box-Cox forms, because the influence of acquisition date is so small.) This implies that actual service lives of buildings have been increasing over time and will continue to do so—a result that surprises me and deserves some attention by the authors.

One of the most widely discussed proposals for depreciation reform is a shift from historical-cost accounting to replacement-cost (or current-cost) accounting. The authors do not address this timely and important issue, and readers might naturally wonder to which accounting scheme the reported depreciation rates pertain. At first blush it appears that the rates should be viewed in the context of historical-cost accounting, since they are to be applied to the depreciated historical costs of assets. This in turn suggests that under inflationary circumstances the rates would be higher or that the reported rates should be applied to the undepreciated balance revalued to the current price level.

However, the situation is not quite so straightforward. Consider, for example, the depreciation of a vintage 1 building during its first year of service. In terms of the notation introduced above, I would define historical-cost depreciation as $Y(0,1) - Y(1,1)$, which measures the effect of aging alone on a building's value. But if the building were sold at the end of its first year of service, it should command a market price of $Y(1,2)$. At the same time, a new building of a similar type should sell for $Y(0,2)$, so that if the owner of the used building were to sell it and replace it with a similar new building, the net cost would be $Y(0,2) - Y(1,2)$. I would define this last expression as replacement-cost depreciation, consistent with the notion that depreciation should allow for the maintenance of *real* wealth. We can now ask under what circumstances the historical-cost and replacement-cost measures will differ. The answer is that they will differ if acquisition date has some influence on value *and* if this influence is not purely additive to the influence of aging.

In the case of the polynomial forms of the price equations, the influence of acquisition date is significant but strictly additive, and we can therefore view the calculated depreciation measures as appropriate to replacement-cost accounting. For the Box-Cox forms, the influence of acquisition date is so small that for all practical purposes it can be ignored. (If acquisition date were significant, its effect would be additive only if θ_1 were unity, which does not appear to be the case.) Thus there is little if any difference between the historical- and replacement-cost measures for these forms. (These statements pertain only to the office-building equations; equations for the other types of buildings are not given in the paper.)

Raising the replacement-cost issue focuses our attention once again on the role of the acquisition-date variables in the estimated value equations. I believe that two points need to be stressed. First, if, as I suggested earlier, these variables are essentially proxies for the general price level, it would be preferable to replace them with an index of prices and not rely on a complex power transformation or polynomial in the acquisition date to capture, in a possibly very crude way, all of the wiggles in the rate of inflation. This would also facilitate extrapolations of the equations into future years when inflation rates might well depart from the sample-period cycles and trends that determine the estimated coefficients on the acquisition-date variables.

Second, it seems unreasonable to introduce either acquisition date or a general price index in the value equations in an additive way. If there is a general inflation occurring that raises the nominal rate of interest but leaves the real rate of return on physical capital unchanged, then the values of all capital goods—new and old—should rise at the prevailing rate of inflation (Coen, 1976). Accordingly, the value equation should have the multiplicative form $Y(S,P_t) = P_t f(S)$, where P_t is a general price index and $f(S)$ is some general expression in age (polynomial, power transform, or whatever). If one wishes to substitute a general expression in acquisition date for P_t, acquisition date would nonetheless appear in a multiplicative way with S. When P_t or its proxy appears additively, the implication is that a given rise in the price level will produce a larger percentage increase in the value of old capital goods than in the value of new capital goods—a phenomenon I can neither justify intuitively nor explain in capital-theoretic terms.

Because of these shortcomings in the Hulten-Wykoff specifications of the value equations, I am reluctant to accept any implications of their estimates regarding the relationships between historical-cost and replacement-cost measures of depreciation.

These specification problems do not apply, of course, to the authors' cross-section estimates of the effects of age alone (that is, to estimates of the $Y(S,t)$ functions in which t is held constant). The cross-section results for office buildings, presented for the first time in this paper, are a very interesting addition to the authors' previous investigations. By and large, they tend to confirm the age effects discovered in the pooled time-series, cross-section estimates discussed above. Depreciation due to aging alone is again generally found to be more accelerated than straight-line and to occur at rates substantially lower than those implied by the 1962 guideline lives. Thus, although the role of acquisition date may be misspecified in the pooled estimates, this difficulty apparently does not lead to serious bias in the coefficients on age in the pooled regressions.

The Hulten-Wykoff cross-section findings are directly comparable to cross-section estimates of actual depreciation of office buildings derived by Taubman and Rasche (1969). The latter authors did not use direct observations of resale prices of buildings but instead studied the age profile of net rents for a sample of office buildings. They computed the present value of net rents for buildings of different ages and defined actual depreciation as the change in present value resulting from aging. Contrary to Hulten and Wykoff's results, Taubman and Rasche found that depreciation was decelerated relative to straight-line. It would certainly be reassuring if these two studies, using different methods and different data sets, arrived at similar conclusions; unfortunately this is not the case.

How do we explain these diametrically opposed results? One possibility is to reject the notion that the value of a building—its market price—bears any systematic relation to the present value of its net rents, i.e., reject the hypothetical market values constructed by Taubman and Rasche. But then what does determine capital values, and how does one explain the negative effects of aging on value discovered by Hulten and Wykoff? I am not ready to forsake this fundamental tenet of capital theory until I have a convincing replacement.

A second possibility is to reject one of the two data sets—perhaps both—as biased. It may be that the used buildings for which Hulten and Wykoff have resale prices are not representative of "the average building." Many of them might, for example, be the remnants of misguided, unprofitable ventures. They might be buildings with very high vacancy rates which had to be liquidated and could be sold only at substantial discounts to be competitive with rates of return being earned on other buildings. If this were the case, the authors would overestimate the depreciation rates on the preponderance of office buildings that seldom appear on secondhand markets. On the other hand, Taubman and Rasche had observations on rents only for rather broad age groups, and their interpolation scheme to obtain a complete rent-age gradient may have been faulty. I must say, though, that the gentle negative slope of their gradient seems very plausible. I doubt that anyone would pay a substantial premium for office space in Chicago's new Sears Tower over similar space in the older John Hancock Building. But even if their rent-age relation is correct, their estimates of operating costs by age, which they use to compute net rents, could be in error.

After some rather unrewarding hours of reflection on this conundrum, I regret having to revert to banality—more research is needed. We need to know more about the resold office buildings in the OIE sample to determine to what extent a process of negative selection generated the observations. We need to initiate new, more complete surveys of office-building rents and operating costs by age of building. We need to combine information on resale prices, rents, and operating costs with building characteristics other than age, such as height and location. To the extent that height and location are correlated with age, their exclusion from price-age or rent-age relations will bias the estimated effects of age.

There are rays of hope here that should not be overlooked. Hulten and Wykoff's findings for factories are roughly consistent with my recent estimates of loss-of-efficiency patterns for manufacturers' structures (Coen, 1974), which I arrived at in a way totally different from either the present authors or Taubman and Rasche. One should not seize on this, however, to make sweeping conclusions about the validity of either Hulten and Wykoff's approach or my own, or about the universality of accelerated economic depreciation. For one thing, I suspect that the nature and importance of obsolescence are quite different for factory and office buildings and the resale markets for the two might be quite dissimilar. (For example, factory buildings are rarely used as "tax shelters," a practice that has not been uncommon in the case of office buildings.) For another thing, I am still at work trying to improve my method, and I do not yet know whether the outcome of my current refinements will be more in line with my initial findings (Coen, 1975), which are not consistent with those presented by Hulten and Wykoff, or with my more recent ones (Coen, 1974).

I believe that economists are still a long way from having definitive estimates of actual depreciation. Various approaches are beginning to yield some common results, but there are still many instances of opposite findings. And even if agreement could be reached on service lives and loss-of-efficiency patterns, there would remain controversial aspects of replacement-cost accounting that need to be resolved.

Finally, and without intending to downgrade the importance of historical investigations of depreciation rates for broad classes of assets, I think we should keep in mind that no general tax depreciation formula is likely to do justice to the "facts and circumstances" of individual firms or taxpayers—a point well known to Treasury agents in the field but seldom appreciated by scholars. Perhaps scholars should begin to devote more of their research ingenuity to devising and perfecting such self-policing accounting mechanisms as the defunct reserve-ratio test, which could provide more or less automatic checks on whether what taxpayers claim to be true is in fact true. This might well turn out to be a more promising route to ensuring that tax and economic depreciation will someday be equated.

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