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EXTRACTING INFORMATION FROM DIFFERENT EXPECTATIONS

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Extracting Information from Different Expectations

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Abstract

Long-term expectations are believed to play a crucial role in driving future inflation and guiding monetary policy responses. However, expectations are not directly observed and the available measures often present a wide range of values. To understand what drives these differences, we examine the evolution of survey and market-based consumer price inflation expectations in the United States between 2003-2019. We show that inflation forecasts can be improved by incorporating the differential between survey and market-based measures. Next, we decompose and extract the differentials in rigidity and information between these measures. While both information and rigidities play a role, the information differential is more important. Using machine learning methods, we find that up to half of the information differential is explained by real-time changes in measures of liquidity. This also explains some past forecast improvements and helps predict the divergence in long-term inflation expectations in 2020.

Keywords: Breakeven inflation, error correction, forecast encompassing, model selection

JEL classifications: C32, C52, C53, D84, E31, E37

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1 Introduction

Long-term inflation expectations are believed to play an important role in driving and predicting future inflation. However, expectations are difficult to measure and there are many ways to do so. Alternative methods, which include surveys (consumers, businesses, forecasters, etc.), financial-markets, and models thereof, produce a wide range of results. For example, since 2014 the Federal Reserve Bank of New York has conducted surveys of primary dealers' and market participants' expectations about 5-year, 5-year forward CPI inflation. Market participants' median expectations have never exceeded and on average are 0.12 percentage points lower than primary dealers (with a t-statistic greater than 10). More generally, differences were also pronounced during the initial spread of COVID-19 in 2020 when market-based measures declined sharply, surveys of professional forecasters were broadly unchanged, and surveys of consumers increased.

The dispersion between survey and market-based measures of inflation expectations raises important questions about the relative informativeness and whether they can be used to predict future inflation. In this paper, we analyze the differences between survey and market-based measures, their informativeness, and what might be driving them. We start with a forecast-encompassing framework to understand whether the differential between surveys and markets can improve inflation forecasts. Next, we decompose and identify the information and rigidity differentials and use machine learning methods to select those variables that best explain the information differential. Finally, we test whether the selected information matters for improving inflation forecasts and if it captures the recent divergence in expectations.

We find that the differential between survey and market-based expectations can be used to improve inflation forecasts. While the differential is driven by both the rigidity and information channels, the information differential closely captures most of the discrepancies between alternative measures of expectations. The information differential is explained by measures of real-time changes in liquidity including income, money, stocks and reserves. We show that changes in liquidity can explain most of the forecast improvements due to differences between survey and market-based expectations and use this to predict much of the divergence in expectations during the spread of COVID-19 in 2020.

Our findings relate to the literature comparing alternative measures of inflation expectations, which was sparked by Bernanke (2007) when he asked “which measure or combination of measures should central bankers focus to assess inflation developments?”.¹ The initial response was in favor of surveys of forecasts from professional economists (see Ang et al., 2007) which were then incorporated into models to improve their forecasts; e.g. see Clark (2011), Kozicki and Tinsley (2012), Faust and Wright (2013), and Chan

¹See also Thomas (1999) for an earlier comparison of survey-based measures which favors professional forecasters.

et al. (2018). However, recent findings support using measures of short-term consumer (Coibion et al., 2018 and Chen, 2019) or market-based (Kliesen, 2015, Grothe and Meyler, 2018 and de Mendonça et al., 2020) inflation expectations.² This coincides with a decline in the usefulness of survey-based inflation expectations (see Trehan, 2015 and Berge, 2018) and as a result, policy-makers pay attention to several measures of inflation expectations albeit with different emphasis (see Bullard, 2016, Yellen, 2017, Böninghausen et al., 2018, and Clarida, 2020). We build on this literature by embedding alternative measures within Faust and Wright (2013)’s autoregressive gap model of inflation and show that long-term market-based measures can improve upon professional survey-based forecasts.

The analysis is also related to the literature on deviations from full information rational expectations, which considers the role of rigidities (Mankiw et al., 2003) and/or noisy information (Sims, 2003 and Woodford, 2003). Unlike previous studies, that test for evidence of these channels in individual and/or aggregate professional forecasters (see Coibion and Gorodnichenko 2015a, Coibion et al. 2018, Bordo et al. 2018 and Angeletos et al. 2020), we consider the differential between aggregate survey and market-based measures. We allow for different information and rigidities across measures and then decompose them to show that the information differential is more correlated with the overall differential. Our approach differs from the decomposition in Reis (2020) by focusing on the disagreement across measures of expectations rather than the disagreement within measures due to heterogeneous agents.

Finally, this analysis also extends the literature on drivers of expectations differentials. Previous studies find that inflation expectations respond differently to news (Carroll, 2003), food and oil prices (Coibion and Gorodnichenko, 2015b), macroeconomic data releases (Bauer, 2015), financial volatility (Stillwagon, 2018), and beliefs (Candia et al., 2020). We take a different approach by extracting the information differential and searching across many potential information sources. We find that the information differential is well explained by real-time changes in liquidity including money, reserves, income, and stocks, which suggests a possible avenue for augmenting models that are used to forecast inflation.

The rest of the paper follows. The next section describes the data and measures of expectations used. Section 3 describes the analytical methods while section 4 presents the results. Finally, section 5 concludes.

2 Measures of Inflation and Inflation Expectations

There are multiple measures of inflation expectations derived from surveys in the United States. We focus on measures of long-term expectations that extend at least five-years-ahead. These include the Blue Chip

²In contrast, Bauer and McCarthy (2015) finds short-term market expectations perform worse than professional forecasters.

Economic Indicators (BCEI), the Livingston Survey (LIV), the Survey of Professional Forecasters (SPF), and the University of Michigan’s Survey of Consumers (MSC).³ The long-term BCEI forecasts are updated twice a year in March and October, the LIV forecasts are updated in June and December, the SPF forecasts are updated quarterly and the MSC expectations are updated monthly.

Inflation expectations derived from financial markets are often based on the difference between Treasury Inflation-Protected Securities (TIPS) and nominal Treasury Securities and are referred to as Treasury Breakeven Inflation.⁴ There are, however, several concerns associated with this measure which complicate its interpretation as a measure of expectations. In particular, breakeven inflation includes a possibly time-varying inflation risk premia as well as a liquidity bias due to differences in the respective markets.

The U.S. Treasury produces a measure of Treasury Breakeven Inflation (TBI) which directly addresses the liquidity bias. The main features of this measure are (1) it is based on off-the-run Treasury securities which substantially reduces differences in liquidity across the nominal Treasury and TIPS markets; and (2) it is calculated using spot rates instead of yields to ensure consistency with inflation rates; see Girola (2019). This measure extends back to 2003 and is updated on a monthly basis using daily data. Church (2019) shows that TBI correlates well with future non-seasonally adjusted CPI.

Alternatively, models are used to extract expectations from breakeven inflation. One approach comes from D’Amico et al. (2018, DKW), who use a no-arbitrage pricing model to extract inflation expectations from nominal yields, TIPS yields, and inflation. Their measure is updated monthly by the Federal Reserve Board; see Kim et al. (2019). We also consider two other model-based measures of long-term inflation expectations. Haubrich et al. (2012, HPR) use a GARCH model to extract monthly inflation expectations from surveys of professional forecasters, nominal Treasury yields, and inflation swaps. Their measure is available on a monthly basis since 1982 and is maintained by the Federal Reserve Bank of Cleveland.⁵ Aruoba (2020, ARU) produces a term-structure of inflation expectations based on information from multiple surveys of professional forecasters. This measure is available on a monthly basis since 1998 and is maintained by the Federal Reserve Bank of Philadelphia.

We measure U.S. inflation using the October 2020 vintage of the seasonally adjusted consumer price inflation (CPI-U). We transform this series into a quarterly annualized inflation rate by averaging the price level over the quarter and then taking the log difference multiplied by 400. The sample of interest is from 2003 Q1 - 2019 Q4 over which the measures of long-term inflation expectations are available.

³Long-term measures of expectations are also available from Blue Chip Financial Forecasters and Consensus Economics (excluded due to similarity with BCEI), the New York Federal Reserve’s Surveys of Primary Dealers and Market Participants as well as the Atlanta Federal Reserve’s Business Inflation Expectations (excluded since surveys start after 2010).

⁴TIPS are based on non-seasonally adjusted CPI. Inflation swaps are also available starting in 2004.

⁵Ajello et al. (2019) and Williams (2020) also propose model-based measures of inflation expectations derived from TIPS.

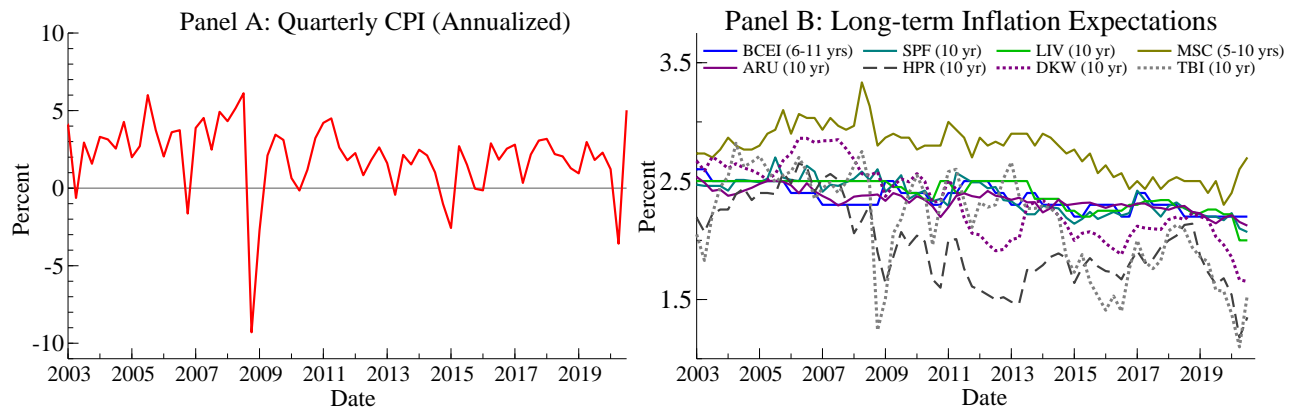


Figure 2.1: Inflation and Long-term Inflation Expectations (2003 Q1 - 2020 Q3)

Figure 2.1 plots inflation in Panel A and the measures of long-term inflation expectations in Panel B. Each measure is presented on a CPI basis with the exception of the Michigan survey which is not tied to a specific measure of inflation. The measures are plotted quarterly from 2003 through 2020 Q3 where higher-(lower-) frequency measures are averaged (extended) across the full quarter.⁶ At the end of 2019, long-term inflation expectations ranged between 1.6 and 2.3 percentage points. There was a clear difference between survey-based measures (indicated by solid lines) and measures derived primarily from financial markets (indicated by dotted or dashed lines). In recent years, survey-based measures were all higher than financial market-based measures. This gap became even more apparent during the spread of COVID-19 in 2020.

Table 2.1: Correlation Across Measures of Expectations (2003-2019)

	BCEI	SPF	LIV	MSC	ARU	HPR	DKW	TBI
BCEI	1.00	0.73	0.67	0.37	0.77	0.29	0.41	0.44
SPF		1.00	0.81	0.60	0.79	0.60	0.74	0.60
LIV			1.00	0.72	0.68	0.41	0.63	0.69
MSC				1.00	0.54	0.42	0.62	0.68
ARU					1.00	0.53	0.54	0.50
HPR						1.00	0.87	0.53
DKW							1.00	0.59
TBI								1.00
PC	0.61	0.85	0.82	0.77	0.74	0.80	0.89	0.84

Notes: Pearson correlation coefficients are presented. PC: First Principal Component of inflation expectations.

We can examine the correlations between measures of inflation expectations to understand their similarities. Table 2.1 presents the correlations across measures and with their first principle component. The measures of expectations derived from professional forecasters are highly correlated with one another and

⁶See Supplemental Material for an illustrative timeline of the release dates for each measure in 2020 Q1.

with the ARU measure. Model-based measures of financial market expectations, HPR and DKW, are highly correlated with one-another despite using different information but are not particularly similar to TBI. Thus, while some measures are similar, there are substantial differences between them, which suggests they may contain unique information. On the other hand, the first principal component of all the measures correlates well with each measure individually. This suggests that there could be a common component or trend. While this is an approach for extracting similar information from multiple measures of expectations, see Ahn and Fulton (2020) and Williams (2020), we focus instead on the differences between measures.

3 Methods

There are several competing explanations for what drives differences in expectations. For example, Mankiw et al. (2003) propose a sticky-information model where gaps in expectations are explained by differences in rigidities. So, for example, market participants might update their expectations more quickly to the same set of information. An alternative explanation comes from a model of noisy-information, see Sims (2003) and Woodford (2003), where agents may have access to different kinds of information. In this context, the gap is explained by access to (or a focus on) different information. Thus, market participants may be responding to a different, perhaps better or worse, information set.⁷

To understand the relevance of these two explanations, it is necessary to establish a framework that can be used to disentangle them. We start with Faust and Wright (2013)’s autoregressive model of the gap between inflation and long-run expectations. The h -step-ahead forecasts from this model can be written as

$$\tilde{\pi}_{t+h|t} = \tilde{\mu}_{t+h|t} + \rho^h (\pi_t - \mu_t), \quad (1)$$

where π_t is inflation and μ_t represents long-term expectations, generated at time t , about future inflation. In the simplest version of (1), the autoregressive parameter is fixed at $\rho \equiv 0.46$.⁸ Future inflation expectations follow a random walk so that $\tilde{\mu}_{t+h|t} = \mu_t$. This implies that (1) has a time-varying long-run mean (commonly referred to as a time-varying trend). Other studies formulate variants of (1) using alternative long-run survey-based professional forecasts or statistical measures.⁹ We are interested if market-based measures can add value beyond survey-based measures and so we appeal to the forecast-encompassing literature.

⁷The Federal Reserve Bank of New York’s October 2015 Survey of Market Participants indicates that both explanations matter.

⁸The parameter for more recent periods is likely smaller than 0.46 as discussed by Chen (2019). Equation (1) nests a special case of the New Keynesian Phillips Curve when assuming the output gap follows an AR(1) process and all other coefficients are unity. Extending (1) to account for the output gap explicitly with fixed parameters produces similar results.

⁹Backwards looking statistical measures were proposed by Atkeson and Ohanian (2001), Stock and Watson (2007) and Ball and Mazumder (2011). Martinez et al. (2021) illustrates why backwards looking measures perform well in some settings.

Suppose for simplicity of exposition that there are only two different measures of expectations whose relative information content are of interest: survey-based expectations $\mu_{t, Survey}$ and financial market-based expectations $\mu_{t, Markets}$. Furthermore, suppose that these measures are used as different time-varying long-run means in (1) to generate alternative forecasts of inflation. Then it is possible to formulate a forecast-encompassing regression as in Chong and Hendry (1986) and Fair and Shiller (1989, 1990):

$$\pi_{t+h} = \beta_{0,h} + \beta_{S,h} \tilde{\pi}_{t+h|t, Survey} + \beta_{M,h} \tilde{\pi}_{t+h|t, Markets} + u_{t+h}, \quad (2)$$

where $\beta_{0,h}$ is the bias and u_{t+h} is the unexplained residual.¹⁰ We can test for the uniqueness of information available from each of the forecasts in this equation.¹¹ For example, the joint hypothesis of $\beta_{S,h} = 1$ and $\beta_{M,h} = 0$ implies that the survey-based forecast sufficiently explains inflation at horizon h so that the market-based forecast provides no additional value. Alternatively, the joint hypothesis of $\beta_{S,h} = 0$ and $\beta_{M,h} = 1$ implies that the survey-based forecast is not informative beyond the market-based forecast.¹²

The equation can be reinterpreted in the context of forecast combinations as originally discussed by Bates and Granger (1969). In fact, as illustrated by Granger and Ramanathan (1984), the population-optimal weights for combining the forecasts can be obtained from (2) when a homogeneity restriction $\beta_{S,h} + \beta_{M,h} \equiv 1$ is imposed. In the context of forecast-encompassing, Ericsson (1993) shows that the homogeneity restriction implies that (2) can be rewritten by subtracting $\tilde{\pi}_{t+h|t, Survey}$ from both sides so that

$$e_{t+h, Survey} = \beta_{0,h} + \beta_{M,h} (\tilde{\pi}_{t+h|t, Markets} - \tilde{\pi}_{t+h|t, Survey}) + \tilde{u}_{t+h}, \quad (3)$$

where $e_{t+h, Survey} = \pi_{t+h} - \tilde{\pi}_{t+h|t, Survey}$ is the survey-based forecast error. The importance of the differential between the market-based forecast and the survey-based forecast is determined by the optimal weight on the market-based forecasts $\beta_{M,h}$ in (3). When both forecasts are generated from (1) with fixed ρ , the forecast differential is the horizon weighted expectations differential

$$(\tilde{\pi}_{t+h|t, Markets} - \tilde{\pi}_{t+h|t, Survey}) = (1 - \rho^h) (\mu_{t, Markets} - \mu_{t, Survey}), \quad (4)$$

where greater weight is given to the expectations differential at longer horizons. Therefore, the focus on the forecast differential is a formalization of the implicit framework by Clarida (2020). In this context,

¹⁰The bias term captures expectation differentials that are constant over time; e.g. see Bürgi (2020).

¹¹The forecasts are non-nested and parameters are fixed so standard t-tests are used (Harvey et al., 1998) without correcting for parameter estimation uncertainty (West, 2001) or nested models (Clark and McCracken, 2001; Hansen and Timmermann, 2015).

¹²Romer and Romer (2000) spurred a separate literature by referring to these as information advantage hypotheses.

the null hypothesis $\beta_{M,h} = 0$ means that the expectations differential provides no explanatory power for the survey-based forecast errors. Alternatively, the null hypothesis that $\beta_{M,h} = 1$ means that the expectations differential completely explains the survey-based forecast errors. Together, both null hypotheses assess whether the differential between market and survey-based measures improves survey-based forecasts.

This framework can be extended in multiple ways. First, it can be used to evaluate two survey-based forecasts. Although the direction of information flow becomes less clear, the approach can still be used to assess whether the expectations differential helps explain the forecast errors. Second, it can be augmented by conditioning on additional information. For example, the unemployment gap can be added to (3) to evaluate if omitting the Phillips curve from the forecasts matters. Third, the forecast-encompassing equation can be extended to test multiple horizons (see Hungnes, 2018) and/or more than two forecasts (see Martinez, 2015 and Ericsson and Martinez, 2019) jointly. Finally, the tests can also be adapted to allow for time variation and instabilities using fluctuation tests; see Rossi and Sekhposyan (2016).

While forecast-encompassing tests evaluate the general difference in information content between the forecasts, they can also be reinterpreted as tests of the combined information and rigidity differentials between measures. To see this, suppose that measures of expectations are described as

$$\mu_{t,m} = \gamma_m f_{t,m} + (1 - \gamma_m) \mu_{t-1,m}, \quad (5)$$

where $f_{t,m}$ is a measure-specific function of information available at time t and γ_m denotes how quickly expectations are updated by this information (i.e. rigidity).¹³ The differential between market-based and survey-based expectations can then be decomposed into the rigidity differential multiplied by the discounted change in survey expectations plus the discounted information differential plus the initial conditions:

$$\begin{aligned} (\mu_{t,Markets} - \mu_{t,Survey}) &= (\gamma_{Markets} - \gamma_{Survey}) \sum_{j=0}^{T-1} (1 - \gamma_{Markets})^j \left[\frac{\Delta \mu_{t-j,Survey}}{\gamma_{Survey}} \right] \\ &\quad + \gamma_{Markets} \sum_{j=0}^{T-1} (1 - \gamma_{Markets})^j [f_{t-j,Markets} - f_{t-j,Survey}] \\ &\quad + (1 - \gamma_{Markets})^T (\mu_{t-T,Markets} - \mu_{t-T,Survey}). \end{aligned} \quad (6)$$

This implies that market-based expectations can be both less rigid, $\gamma_{Survey} < \gamma_{Markets}$, and have access to different information $f_{t,Markets} \neq f_{t,Survey}$ such that a combination of the information and rigidity differentials drives the total expectations differential and therefore the forecast discrepancy.

¹³This is consistent with the earlier assumption that $\mu_{t+h|t}$ is a random walk when $f_{s,t}$ is also treated as a random walk.

We can use the decomposition in (6) to understand which of the two channels is more important for predicting future inflation. However, we first need to identify the degree of information rigidity in each measure of expectations. To do this, we impose a common function of information across each measure of expectations as a biased and noisy prediction of current inflation $f_{t,m} = \eta_m + \pi_t + \varepsilon_{t,m}$ so that (5) becomes

$$\mu_{t,m} = \gamma_m (\pi_t + \eta_m) + (1 - \gamma_m) \mu_{t-1,m} + \gamma_m \varepsilon_{t,m}, \quad (7)$$

which is similar to the approaches by Coibion and Gorodnichenko (2015a) and Jorgensen and Lansing (2019) among others except that we allow for measure specific intercepts and information beyond current inflation to impact long-term expectations formation. For example, Bauer (2015) finds that both market-based and survey-based measures of expectations respond to inflation and macroeconomic news releases.

However, considering (1) and (7) together implies that expectations and inflation are determined simultaneously when $\gamma_m \neq 0$. This means that γ_m cannot be identified from (7) alone. Therefore, we formulate (1) and (7) as a system of simultaneous equations

$$\begin{pmatrix} 1 & -1 \\ -\gamma_m & 1 \end{pmatrix} \begin{pmatrix} \pi_t \\ \mu_{t,m} \end{pmatrix} = \begin{pmatrix} \eta_{\pi,m} \\ \gamma_m \eta_{\mu,m} \end{pmatrix} + \begin{pmatrix} \rho & -\rho \\ 0 & 1 - \gamma_m \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ \mu_{t-1,m} \end{pmatrix} + \begin{pmatrix} v_t \\ \gamma_m \varepsilon_{t,m} \end{pmatrix}, \quad (8)$$

where we assume that the shocks v_t and $\varepsilon_{t,m}$ are mean zero with a general covariance structure. If $\gamma_m \neq 1$, then we can rewrite (8) as a system of reduced form equations

$$\begin{pmatrix} \pi_t \\ \mu_{t,m} \end{pmatrix} = \frac{\gamma_m}{1 - \gamma_m} \begin{pmatrix} \frac{1}{\gamma_m} \eta_{\pi,m} + \eta_{\mu,m} \\ \eta_{\pi,m} + \eta_{\mu,m} \end{pmatrix} + \begin{pmatrix} \frac{\rho}{1 - \gamma_m} & 1 - \frac{\rho}{1 - \gamma_m} \\ \frac{\gamma_m \rho}{1 - \gamma_m} & 1 - \frac{\gamma_m \rho}{1 - \gamma_m} \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ \mu_{t-1,m} \end{pmatrix} + \frac{\gamma_m}{1 - \gamma_m} \begin{pmatrix} \frac{1}{\gamma_m} v_t + \varepsilon_{t,m} \\ v_t + \varepsilon_{t,m} \end{pmatrix}, \quad (9)$$

which can also be expressed in vector equilibrium-correction form

$$\begin{pmatrix} \Delta \pi_t \\ \Delta \mu_{t,m} \end{pmatrix} = \frac{\gamma_m}{1 - \gamma_m} \begin{pmatrix} \frac{1}{\gamma_m} \eta_{\pi,m} + \eta_{\mu,m} \\ \eta_{\pi,m} + \eta_{\mu,m} \end{pmatrix} + \begin{pmatrix} \frac{\rho}{1 - \gamma_m} - 1 \\ \frac{\gamma_m \rho}{1 - \gamma_m} \end{pmatrix} (\pi_{t-1} - \mu_{t-1,m}) + \frac{\gamma_m}{1 - \gamma_m} \begin{pmatrix} \frac{1}{\gamma_m} v_t + \varepsilon_{t,m} \\ v_t + \varepsilon_{t,m} \end{pmatrix}, \quad (10)$$

where when $\gamma_m \neq 0$ weak exogeneity is violated such that there is a reduced-rank restriction that pins down both equations and implies that the system in levels is a non-stationary I(1) process; see Hendry (1995).

We can estimate the bivariate vector equilibrium-correction model in (10) as

$$\begin{pmatrix} \Delta \hat{\pi}_t \\ \Delta \hat{\mu}_{t,m} \end{pmatrix} = \begin{pmatrix} \hat{c}_{\pi,m} \\ \hat{c}_{\mu,m} \end{pmatrix} + \begin{pmatrix} \hat{\alpha}_{1,m} \\ \hat{\alpha}_{2,m} \end{pmatrix} (\pi_{t-1} - \mu_{t-1,m}), \quad (11)$$

where $\hat{\alpha}_{1,m}$ and $\hat{\alpha}_{2,m}$ are the estimates of $\alpha_{1,m} = \frac{\rho}{1-\gamma_m} - 1$ and $\alpha_{2,m} = \frac{\gamma_m \rho}{1-\gamma_m}$ and where $\hat{c}_{i,m}$ are the variable specific intercepts. To identify γ_m from (11), assuming ρ is known, we impose the restriction that $\hat{\alpha}_{2,m} = \hat{\alpha}_{1,m} + 1 - \rho$. Then $\hat{\gamma}_m = 1 - \frac{\rho}{\hat{\alpha}_{1,m} + 1}$. Assuming the residuals are i.i.d. normal then, when applying the multivariate delta method (Casella and Berger, 2002), the coefficient standard error is $\hat{\sigma}_{\hat{\gamma}_m} = \frac{\rho \hat{\sigma}_{\hat{\alpha}_{1,m}}}{(\hat{\alpha}_{1,m} + 1)^2}$.¹⁴

Once we identify the degree of information rigidity for each measure of expectations, it is possible to decompose differences in expectations using (6). Shutting down the differences in rigidities by setting $\gamma_{Markets} \equiv \gamma_{Survey}$ while conditioning on the original set of information or shutting down the information channel by setting $f_{t,Markets} \equiv f_{t,Survey}$ can help illustrate the roles that information and rigidity play in generating a wedge between alternative measures of expectations.

We also examine if economic variables can explain the information differential. To do this, we formulate a model based on the extracted information differential for each measure of expectations

$$\nabla f_{t,m} = (f_{t,m} - f_{t,BCEI}) = \beta_{0,m} + \sum_{i=1}^N \beta_{i,m} \Delta x_{i,t} + o_{t,m}, \quad (12)$$

where $\Delta x_{i,t}$ represents the percent change in a source of information at time t and $o_{t,m}$ is the unexplained residual. When $N > T$, it is not possible to estimate (12) using traditional methods. We select over all N variables using the general-to-specific automatic model selection procedure implemented in ‘Autometrics’; see Doornik (2009). Autometrics performs a tree search over subsets of variables and uses F-tests to eliminate them in groups. It then checks the selected model against the starting point to see if the user-specified amount of information loss is exceeded. This reduces concerns about highly correlated variables that plague other model selection procedures; see Doornik (2008) and Hendry and Doornik (2014). Once we isolate the most important variables, we can use them to predict the information differential, understand whether they drive the overall differential, and ascertain to what extent they explain historical forecast improvements.

4 Results

We start by testing the overall information differences in individual forecast pairs. The forecast-encompassing test results are presented in Table 4.1 where major columns represent different horizons and major rows represent different measures of expectations. For simplicity we choose BCEI as the baseline and focus on the MSC and market-based measures as alternatives. In each block of cells, the first entry presents the estimate of $\beta_{M,h}$ from (3). The second entry is the p-value associated with the null hypothesis that $\beta_{M,h} = 1$. The third entry is the p-value associated with the null hypothesis that $\beta_{M,h} = 0$.

¹⁴The i.i.d. assumption can be relaxed by estimating $\hat{\sigma}_{\hat{\alpha}_{1,m}}$ using HAC methods; see Andrews (1991).

Table 4.1: Forecast-Encompassing Coefficients and Probabilities Relative to BCEI

Horizon:	1	2	3	4	5	6	7	8	Joint
MSC	3.12 [[0.261]] {0.100}	-0.91 [[0.390]] {0.681}	-1.30 [[0.234]] {0.499}	-0.11 [[0.442]] {0.939}	0.87 [[0.916]] {0.470}	0.13 [[0.515]] {0.925}	-1.50 [[0.075]]* {0.282}	-1.27 [[0.063]]* {0.294}	-2.31 [[0.249]] {0.417}
HPR	0.83 [[0.872]] {0.421}	0.72 [[0.794]] {0.494}	1.73 [[0.350]] {0.029}**	2.49 [[0.033]]** {0.001}***	2.59 [[0.028]]** {0.001}***	1.97 [[0.271]] {0.028}**	1.57 [[0.585]] {0.137}	1.46 [[0.650]] {0.155}	1.67 [[0.310]] {0.008}***
DKW	2.00 [[0.361]] {0.070}*	1.96 [[0.322]] {0.045}**	2.20 [[0.162]] {0.012}**	2.49 [[0.076]]* {0.004}***	2.35 [[0.189]] {0.017}**	1.82 [[0.449]] {0.095}*	1.62 [[0.582]] {0.152}	1.30 [[0.778]] {0.225}	1.45 [[0.904]] {0.053}*
TBI	0.10 [[0.372]] {0.918}	-0.52 [[0.139]] {0.608}	-0.20 [[0.169]] {0.816}	0.58 [[0.508]] {0.356}	1.25 [[0.677]] {0.039}**	0.62 [[0.519]] {0.285}	-0.42 [[0.020]]** {0.484}	-0.99 [[0.006]]*** {0.158}	-0.49 [[0.338]] {0.436}

Notes: Horizon is number of quarters-ahead that are being forecast. The values in each block are 1): the estimated coefficients from equation (3) with a dummy variable for 2008 Q4, 2): The p-value associated with the null-hypothesis that the coefficient is equal to unity in the square brackets; and 3): The p-value associated with the null hypothesis that the coefficient is equal to zero in the squigly brackets. All tests use HAC estimates from Andrews (1991). * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

For example, in the third major row and the first major column, the estimate for the optimal weight on DKW at $h = 1$ is $\hat{\beta}_{DKW,1} = 2$. The homogeneity restriction implies that the optimal weight on BCEI is therefore $\hat{\beta}_{BCEI,1} = -1$. The null hypothesis that $\beta_{DKW,1} = 1$ cannot be rejected with a probability of 36.1 percent. However, the null hypothesis that $\beta_{DKW,1} = 0$ is weakly rejected with a probability of 7.0 percent. Together, these results suggest that the expectations differential between DKW and BCEI explains the BCEI-based forecast-errors at this horizon.

The forecast-encompassing results are generally supportive of market-based measures of expectations. Both the HPR and the DKW differentials are informative across several horizons with DKW doing particularly well up through six-quarters-ahead. Furthermore, both the HPR and DKW differentials are informative for BCEI when considering all horizons jointly. The evidence is mixed for the TBI differential in that it is informative at the 1-year-ahead forecast horizon but is not informative at longer horizons. These results are robust to the use of alternative survey-based measures as a baseline and are broadly robust to controlling for the unemployment gap as a measure of the Phillips curve; see Supplemental Material. Following Ericsson (1992), the forecast encompassing results are also mirrored by the relative RMSE rankings; see Supplemental Material. The results are somewhat sensitive to time-varying instabilities, especially for MSC and TBI, and most of the informative differences appear to be significant around 2010-12; see Supplemental Material.

Overall, the results for HPR and DKW indicate that the differential between the survey (BCEI) and market-based (HPR and DKW) measures of expectations can help (and in some cases completely) explain survey-based forecast errors. This supports the hypothesis that market-based measures have access to better

information and/or respond more quickly than survey-based measures to the same information. However, the results are unable to shed light on which of these two channels may be most important for understanding why the differential generates forecast improvements.

4.1 Identifying Rigidity

We now decompose the differences between alternative measures of expectations in order to better understand what is driving them and where the relative value stems from. We start by estimating the degree of information rigidity for each measure of expectations using (11) .

The estimates of γ_m are shown in Table 4.2. The first row presents the baseline estimates assuming the model is correctly specified and without allowing for any instabilities. Many of the estimates are not significantly different from zero. This implies that expectations are weakly exogenous with respect to inflation and so follow a random walk process. However, the HPR measure of expectations has estimates which are positive and significantly different from zero. This implies that it is equilibrium-correcting such that it adjusts to reduce the gap with past inflation. In particular, the estimate of 0.025 for HPR implies that it adjusts back to the equilibrium relationship within 10 years. If the BCEI and TBI estimates were statistically significant, then the negative values would imply that they are equilibrium diverging such that they adjust to increase the gap with past inflation.

	BCEI	MSC	HPR	DKW	TBI
No Model Instability:	-0.005 (0.007)	0.005 (0.012)	0.025* (0.015)	0.013 (0.010)	-0.023 (0.025)
Model Instability / Misspecification:	0.019** (0.008)	0.007 (0.011)	0.025* (0.015)	0.013 (0.010)	-0.031 (0.024)

*Notes: All equations are estimated from 2003Q2 - 2019Q4 (67 observations) with a constant. Allow for model instability/misspecification by selecting over and retaining potential outliers and shifts using Autometrics with Impulse Indicator Saturation (IIS) and Differenced-IIS (DIIS). The target gauge is 0.1% so that under the null hypothesis we expect 0.2 irrelevant indicators to be retained on average. See Supplemental Material for the retained impulses. The estimated standard errors are in parentheses. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$*

We also allow for model instability or misspecification. This can have a large impact on how important expectations are for inflation; see Castle et al. (2014). We allow for any number of outliers and shifts at any point in time by re-estimating (11) with Impulse Indicator Saturation (IIS) and Differenced-IIS (DIIS) using a target gauge of 0.1% (see Hendry et al., 2008 and Castle et al., 2011). 2008 Q4 is retained as a significant outlier across all measures. Table 4.2 shows that when accounting for instabilities, the estimate for BCEI goes from being weakly exogenous to equilibrium-correcting while the estimate for TBI declines

but remains statistically insignificant, which is shown in the recursive plots in the Supplemental Material. There is a concern that the BCEI's lower frequency update schedule could affect its estimates. However, the Supplemental Material shows similar estimates for higher frequency surveys of professional forecasters.

The estimates of γ_m imply that the inertia in long-run expectations was very high over this period regardless of which measure is used. This finding is consistent with other studies that show persistence has increased in the past few decades; e.g. see Jorgensen and Lansing (2019). Note that the estimates of γ_m are somewhat sensitive to the choice of ρ . For example, if inflation persistence has declined to $\rho = 0.2$, as suggested by Chen (2019), then the estimates of γ_m for the survey-based measures would be slightly larger, whereas the estimates for the market-based measures would be somewhat smaller.

4.2 Rigidity vs. Information Channels

The contributions from the various components of the expectations differentials are plotted in Figure 4.1 for four measures. This is based on the decomposition in (6) and using the estimates for γ_m when allowing for model instabilities in Table 4.2.¹⁵ We start by plotting the overall differences between BCEI and four other measures. Next, we shut down the differences in the information channel so that, $f_{t,m} = f_{t,BCEI}$, which implies that only the differences in rigidity and the initial conditions are allowed to operate. Figure 4.1 shows that this explains very little of the overall differentials, with the possible exceptions of MSC and TBI.

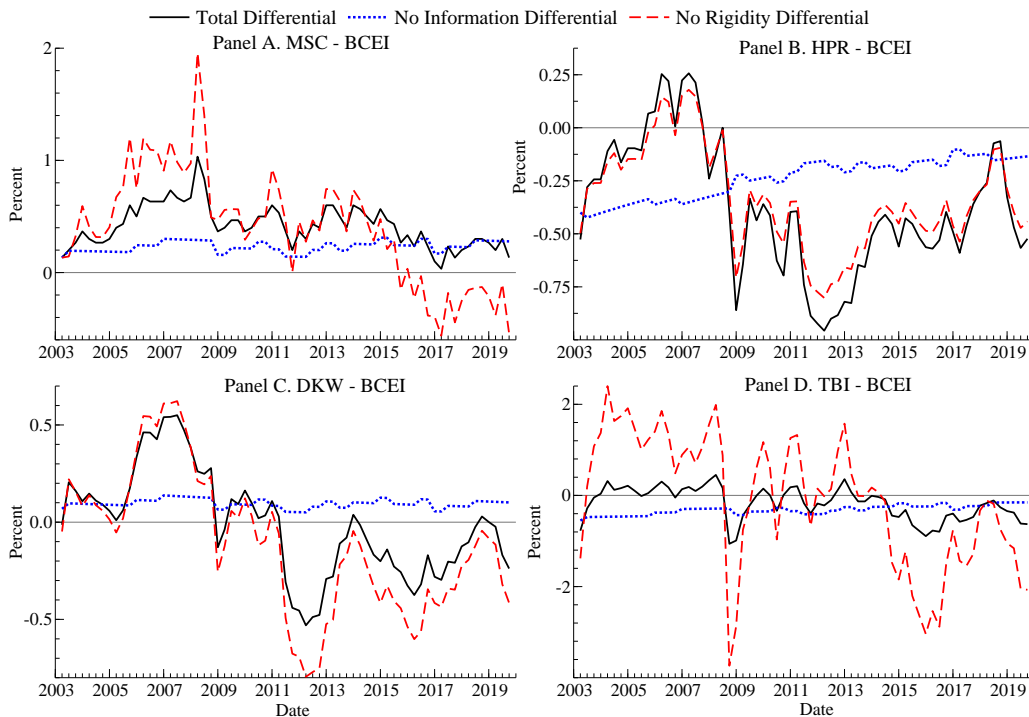


Figure 4.1: Sources of Expectations Differentials by Measure (2003 Q2 - 2019 Q4)

¹⁵Note that we set $\gamma_{TBI} = 0.005$ in line with the smallest other estimate to ensure that its estimated value is non-negative.

Finally, we close the rigidity differential, such that $\gamma_m = \gamma_{BCEI}$, which means that only the information differential and the initial conditions are allowed to operate. Figure 4.1 shows that the information differential captures most of the overall differential. Despite the seemingly small impact of accounting for model instability in Table 4.2, if we ignore the outliers in 2008 Q4 and 2009 Q1, we would mistakenly conclude that the rigidity differential captures most of the overall differential relative to BCEI. This is because at the onset of the Great Recession, the BCEI responded by briefly raising expectations as inflation was falling, which suggests that it was equilibrium-diverging from inflation. Thus, changes in the rigidity estimates can have meaningful impacts on the decomposition of the information and rigidity differentials.

We also detect some other interesting measure-specific features. For example, the total differential between MSC and BCEI and the information differential follow each other closely until 2015 when the information differential declines and the rigidity differential becomes more important. This is consistent with previous findings that recent changes in consumer expectations are not driven exclusively by information (Vergbrugge and Binder, 2016) and may be explained by changing survey demographics as older participants leave the sample; e.g. see Malmendier and Nagel (2016) and Binder and Makridis (2020).

4.3 What Explains Differences in Information?

Previous studies find that inflation expectations respond to a variety of information sources including news (Carroll, 2003), food and oil prices (Coibion and Gorodnichenko, 2015b), macroeconomic data releases (Bauer, 2015), and financial volatility (Stillwagon, 2018). We build on this literature by taking a more general approach and consider every non-interest or exchange rate series available in all vintages of FRED-MD between 2003-2019 as a potential source of information that could produce a wedge between measures of expectations; see McCracken and Ng (2016). This allows us to both assess whether previous findings are corroborated while also potentially discovering other sources of information that have not yet been considered to be important.

For each of the 103 variables that meet our criteria, we aggregate the available data over the quarter and then compute the quarterly percent change for each vintage of the database that was available at the end of each quarter from 2003 Q1 through 2019 Q4. We start by formulating a general unrestricted model as in (12) and then choose a conservative target gauge of 0.1% so that under the null, when selecting over all 103 variables in FRED-MD, on average we expect to retain less than 0.1 irrelevant variables by chance. This ensures that we only retain those variables that matter most for explaining the information differential. Given its importance, following Coibion and Gorodnichenko (2015b), we force oil prices into the model so that they are always retained.

Table 4.3: What Explains Information Differences with BCEI?

Source	∇f_{MSC}	∇f_{HPR}	∇f_{DKW}	∇f_{TBI}
Forced:				
Oil Prices	x	x	x	x
Selected:				
PPI: Crude Materials	+			
Real Money Stock (M2)		-		
Real Personal Income	+			
Real Personal Income (excl. transfers)	-			
Reserves in Depository Institutions				-
S&P 500 Index: Composite			+	
S&P 500 Index: Industrials			-	
R^2 :	0.34	0.33	0.30	0.49

The selection results are presented in Table 4.3, which list the sources retained for each expectations differential, the sign of the relationship, and the fit of the model. A handful of variables can explain between a third and a half of the information differentials. There is limited overlap between the variables retained across the various measures. Therefore, we interpret this to be capturing the differences in idiosyncratic shocks after removing the common information. For example, the idiosyncratic differential for MSC is positively related to changes in government transfers (i.e. the net difference between personal income and personal income excluding transfers) and the price of crude materials. This suggests that consumer expectations are largely associated with the business cycle and changes in prices of food and gas.

The market-based idiosyncratic differentials are influenced by changes in measures of liquidity. The HPR differential is negatively related to changes in the real money stock, the DKW differential is negatively related to changes in the industrials equity price gap, and the TBI differential is negatively related to changes in bank reserves. The negative relationships between measures of liquidity and the market-based differentials suggests that markets interpreted changes in liquidity over this period as being in response to sharp declines in the velocity of money and so lowered their long-term inflation expectations.

These results are consistent with the fact that the sample is dominated by the 2009 financial crisis and its aftermath. During this period there were large-scale expansions of the money supply due to the Federal Reserve's credit and Quantitative Easing programs, a surge in excess bank reserves and large swings in equity prices. All of these are indicative of structural changes in the economy that helped push expected inflation downwards. Note that while the results for HPR and DKW are robust to treating 2008 Q4 as an outlier, the selected information sources for TBI (and MSC) are sensitive to this determination and to changes in the selection procedure; see Supplemental Material.

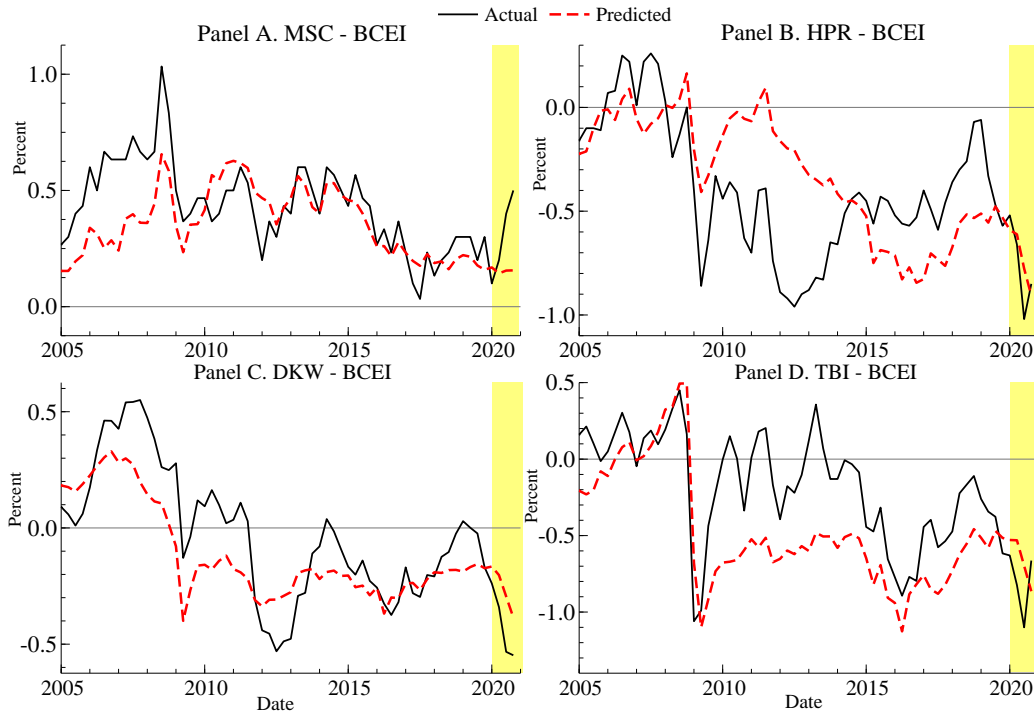


Figure 4.2: Actual and Predicted Expectations Differentials (2005 Q1 - 2020 Q3)

To understand if changes in liquidity continue to matter out-of-sample and if they explain the divergence in early 2020, we use the selected information to project the information differential through 2020 Q3 and then feed both the estimated and the projected differential through (6) to obtain estimates and predictions of the total expectations differential. Figure 4.2 presents the results from this exercise. The selected information does not capture the recent increase in consumer expectations, which is consistent with the fact that the COVID-19 induced recession and the policy response to it has had a different impact on consumers than the 2009 financial crisis.¹⁶ However, the selected information does broadly capture the decline in market-based expectations. The out-of-sample fit, based on the three quarters of 2020 and measured by the root mean square forecast error, is better than the in-sample-fit for both HPR and TBI and roughly the same for DKW.

4.4 Do Changes in Liquidity Explain Better Forecast Performance?

We have seen that changes in liquidity can explain many important fluctuations in the expectations differential. However, we now assess whether changes in liquidity are what drive differences in expectations and improvements in forecasts of future inflation. To do this, we extend the forecast encompassing exercise to isolate the sources of forecast improvements. We augment the transformed equation (3) with the estimates of the information and rigidity channels and perform the encompassing tests conditional on these channels.

¹⁶Note that it is possible to capture the increase in expectations by choosing a looser selection target over the historical sample. However, the economic interpretation becomes more convoluted when many additional variables are selected.

We start by conditioning on the expectations differential when the information channel is closed ($\nabla f_{t,m} \equiv 0$). Thus, the forecast encompassing test becomes a direct test of the value of the information differential for predicting future inflation. Next, we condition on the expectations differential when the rigidity channel is closed ($\gamma_m \equiv \gamma_{BCEI}$) and only the predicted information differential ($\nabla f_{t,m} \equiv \nabla \hat{f}_{t,m}$) operates as generated from Table 4.3. Now the forecast encompassing tests focus on whether controlling for the predictable information differential improves the forecasts. Taken together these tests assess whether the information channel drives improved forecast performance and also whether we have captured those differences using measures of liquidity and oil prices as information sources.

Table 4.4: Augmented Forecast-Encompassing Tests (Relative to BCEI)

h	HPR			DKW		
	(1): Total	(2): (1) $\nabla \gamma_{HPR}$	(3): (1) $\nabla \hat{f}_{HPR}$	(4): Total	(5): (4) $\nabla \gamma_{DKW}$	(6): (4) $\nabla \hat{f}_{DKW}$
1	0.83 [[0.872]] {0.421}	-0.36 [[0.318]] {0.791}	2.30 [[0.235]] {0.040}**	2.00 [[0.361]] {0.070}*	0.66 [[0.798]] {0.626}	1.99 [[0.621]] {0.322}
4	2.49 [[0.033]]** {0.001}***	1.90 [[0.216]] {0.001}***	3.57 [[0.003]]*** {0.000}***	2.49 [[0.076]]* {0.004}***	2.16 [[0.219]] {0.024}**	0.69 [[0.840]] {0.649}
8	1.46 [[0.650]] {0.155}	0.852 [[0.909]] {0.511}	2.17 [[0.231]] {0.029}**	1.30 [[0.778]] {0.225}	2.45 [[0.290]] {0.076}*	0.25 [[0.639]] {0.982}
Joint	1.67 [[0.310]] {0.008}***	0.21 [[0.738]] {0.463}	3.68 [[0.007]]*** {0.000}***	1.45 [[0.904]] {0.053}*	2.07 [[0.702]] {0.364}	-0.86 [[0.975]] {0.981}

Notes: h represents the number of quarters-ahead that are being forecast. The values are the estimated coefficients from equation (3) with a dummy variable for 2008Q4. The Joint estimates and tests follow as a special case from Hungnes (2018) and are estimated with a dummy variable for 2008Q4. The p -value associated with the null-hypothesis that the coefficient is equal to unity is in the square brackets. The p -value associated with the null hypothesis that the coefficient is equal to zero is in the squigly brackets. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

The results are presented in Table 4.4. We focus on four forecast horizons and the two measures of expectations that are able to consistently improve upon the BCEI-based inflation forecasts: HPR and DKW. Columns (1) and (4) replicate the results for HPR and DKW respectively in Table 4.2. Columns (2) and (5) test the importance of the information channel controlling for the rigidity differential. Columns (3) and (6) test whether other information and / or channels matter by controlling for the predicted information differential. The overall results are somewhat mixed with evidence that the rigidity and information differentials both played a role. For HPR, contrary to what we might expect, controlling for the predicted information seems to strengthen the encompassing results. This is due to the fact that the predicted information differential misses the post-2008 decline in the information differential; see Figure 4.2. For DKW, the results are generally consistent with the hypothesis that the information channel matters and changes in liquidity are driving this dynamic. This is demonstrated by the fact that after controlling for the rigidity differential in

column (5), the evidence of forecast-encompassing becomes stronger for the 1-year and 2-year-ahead forecast horizons. At the same time, there is no longer evidence of any information advantage once we control for the predicted differences in information in column (6). Thus, our results show that changes in liquidity explain some of the forecast improvements due to differences in expectations.

5 Conclusions

Long-term inflation expectations are believed to play an important role in driving and predicting future inflation. However, inflation expectations are not directly observed and the various measures derived from surveys of professional forecasters, consumers, and financial markets present a wide range of values. This raises questions as to whether the difference between survey and market-based measures contains information that can be used to forecast inflation.

We start with a forecast-encompassing framework to understand whether the differential between survey and market-based measures is informative for forecasts. Next, we decompose and identify the information and rigidity differentials and use machine learning methods to select over many possible variables that best explain the information differential. Finally, we test whether the selected information matters for improving inflation forecasts and if it captures the recent divergence in expectations.

Applying the methods to multiple measures of CPI inflation expectations in the United States since 2003, we find that the differential between survey and market-based measures adds value to forecasts derived from survey-based measures. We identify the degree of rigidity in each measure of expectations using a constrained bivariate equilibrium correction model and use these estimates to decompose the overall differential into its relative contributions. We find that although the rigidity and information differentials both play a role, the information differential is more closely correlated with the overall differential.

The information differential is explained by a handful of variables which correspond with broad measures of liquidity. We show that these changes in liquidity help predict the divergence between expectations in 2020 and drives much of the improvements in forecast performance due to differences between survey and market-based measures. Overall, our findings illustrate that market-based measures of expectations include a unique information set and that this information can be used to augment existing models of inflation in order to improve their forecast performance.

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A Supplemental Material

A.1 MSE's

Ericsson (1992) notes that a smaller MSFE is a necessary but not sufficient condition to ensure that one forecast encompasses the other. The full sample forecast performance presented in Table A.1 illustrates that the differences between BCEI and most measures are generally not very large. MSC and LIV perform significantly worse, particularly at longer horizons. HPR and DKW perform slightly better although only HPR is significantly better around 1-year-ahead. TBI is inconsistent in that it performs significantly worse at the shortest and longest horizons but is significantly better around 1-year-ahead. ARU also performs significantly better at 1-year-ahead.

Table A.1: Relative Forecast Performance of Alternative Measures of Expectations

h	RMSE	Relative RMSE (in %)								
	BCEI	SPF	LIV	MSC	ARU	HPR	DKW	TBI	AVE	PC-F
1	2.34	100.2	100.1	101.8	100.2	100.3	99.9	100.9	100.2	100.2
2	2.44	100.5	100.5	104.6	100.3	99.8	99.6	102.7	100.6	100.6
3	2.35	100.4	100.6	105.3	100.2	97.5	99.4	102.2	100.2	100.2
4	2.35	99.8	100.4	105.0	99.7	96.7*	99.2	100.0	99.5	99.6
5	2.34	99.8	100.6	104.8	99.4*	97.1	99.6	98.0	99.3	99.4
6	2.33	100.2	101.3	105.7	99.9	98.3	100.2	99.6	100.0	100.1
7	2.32	100.4	101.4	106.5*	100.4	98.9	100.5	102.1	100.7	100.6
8	2.33	100.8	101.1	106.5*	100.2	98.2	100.4	103.1	100.7	100.7
Joint	2.34	100.2	100.6*	103.6*	99.9	98.0	100.0	100.8	100.2	100.2

Notes: *h* represents the number of quarters-ahead that are being forecast. AVE is the average of the different forecasts while PC-F is the First Principal Component of the forecasts following Hillebrand et al. (2018). The joint metric is a generalized version of the RMSE; see Clements and Hendry (1993). Tests of equal predictive accuracy are conducted using Diebold and Mariano (1995) and for joint horizons using Martinez (2017) where stars indicate a rejection of the null hypothesis of equal accuracy with the following probabilities: * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

The differences in relative forecast performance are larger when focusing on the last ten years. This is illustrated in Table A.2 which shows that MSC performs significantly worse whereas the HPR performs better across all horizons. The path measures also suggest that SPF, ARU, HPR and DKW outperform relative to BCEI whereas LIV and MSC underperform relative to BCEI. Overall HPR outperforms across most metrics while MSC underperforms. However, MSC's underperformance is most likely due to inconsistencies in the measure of inflation which the MSC targets; see Bürgi (2020). The performance of measures such as SPF, ARU, DKW and TBI is less consistent across horizons.

Table A.2: Relative Forecast Performance of Alternative Measures of Expectations since 2010

h	RMSE	Relative RMSE (in %)								
	BCEI	SPF	LIV	MSC	ARU	HPR	DKW	TBI	AVE	PC-F
1	1.37	99.8	99.6	104.7	99.7	99.5	99.0	101.1	99.7	99.7
2	1.52	100.1	100.7	109.4*	100.2	96.7	98.5	101.6	99.9	99.9
3	1.51	99.8	101.1**	113.0*	99.6	93.3	97.6	99.1	99.2	99.3
4	1.51	98.6	100.6*	113.5*	98.6	89.9	96.1*	95.7	97.6**	97.8**
5	1.52	97.9	100.8	113.4*	98.1*	88.7	95.9*	94.5	97.0**	97.2**
6	1.51	99.7	103.1*	115.4*	99.2	89.3	96.9	98.4	98.6	98.7
7	1.50	101.4	102.9	118.2*	100.2	90.8	97.3	105.7	100.4	100.4
8	1.51	101.6	102.7*	118.5**	100.3	92.7	98.3	111.5*	101.5	101.5
Joint	1.36	99.7*	100.5*	103.9	99.7	95.4	98.4	100.3	99.5	99.5

Notes: h represents the number of quarters-ahead that are being forecast. AVE is the average of the different forecasts while PC-F is the First Principal Component of the forecasts following Hillebrand et al. (2018). The joint metric is a generalized version of the RMSE; see Clements and Hendry (1993). Tests of equal predictive accuracy are conducted using Diebold and Mariano (1995) and for the joint horizons using Martinez (2017) where stars indicate a rejection of the null hypothesis of equal accuracy with the following probabilities: * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

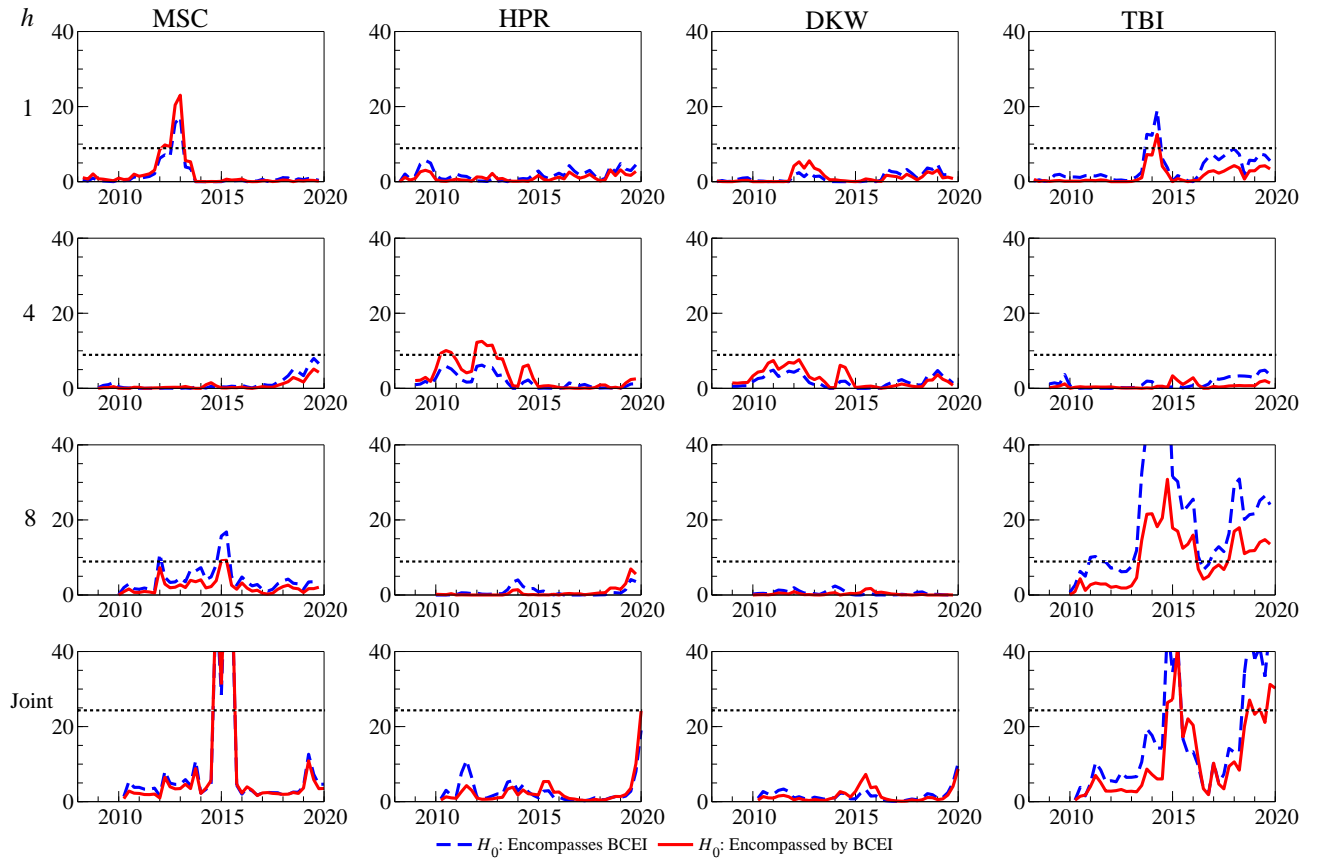
A.2 Fluctuation Encompassing Tests

We can assess the stability of the encompassing tests over time by re-estimating the encompassing test statistics based on (3) using a 20 quarter (five year) rolling window. Since this is a restricted version of Hoesch et al. (2020)'s information-advantage fluctuation regression, the critical values from Rossi and Sekhposyan (2016) can be used.¹⁷ We focus on four horizons: $h = \{1, 4, 8, \text{Joint}\}$ and four measures of expectations: MSC, HPR, DKW and TBI. The recursive test statistics are presented in Figure A.1.

There is strong evidence of instability in the encompassing test results for MSC and for TBI. This is especially true for the 1-quarter-ahead, 2-years-ahead, and the joint forecast horizons. For MSC, this occurs around the middle of the sample around 2012-2015 which coincides with the start of the post-2008 sample and could indicate underlying instabilities in MSC at that time. The instability of TBI is associated with the latter half of the sample around 2015 and 2018-19 where both hypotheses are strongly rejected.

The results for HPR and DKW are relatively stable with neither hypothesis is rejected except for a brief period around 2010-12 at the one-year-ahead horizon where both HPR and DKW are more likely to encompass BCEI. There is also evidence towards the end of the sample that HPR is more informative than BCEI for all forecast horizons jointly. This indicates that the optimal weights are unstable over time.

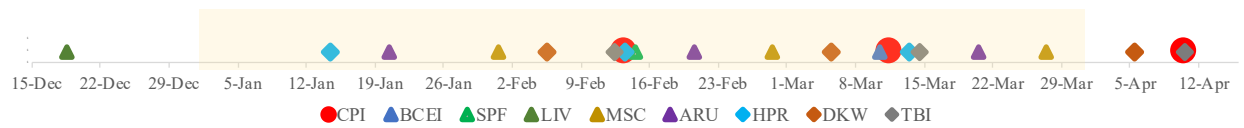
¹⁷We use the model-free critical values. Rossi and Sekhposyan (2016) argue that these critical values are valid across all horizons.



Notes: Each regression tests a single restriction (8 restrictions for the joint) and the estimation window of 20 quarters represents about 30% of the overall sample. All statistics are computed using HAC estimates from Andrews (1991). The dotted black line represents the critical value at which the null hypothesis is rejected at a 5% confidence level.

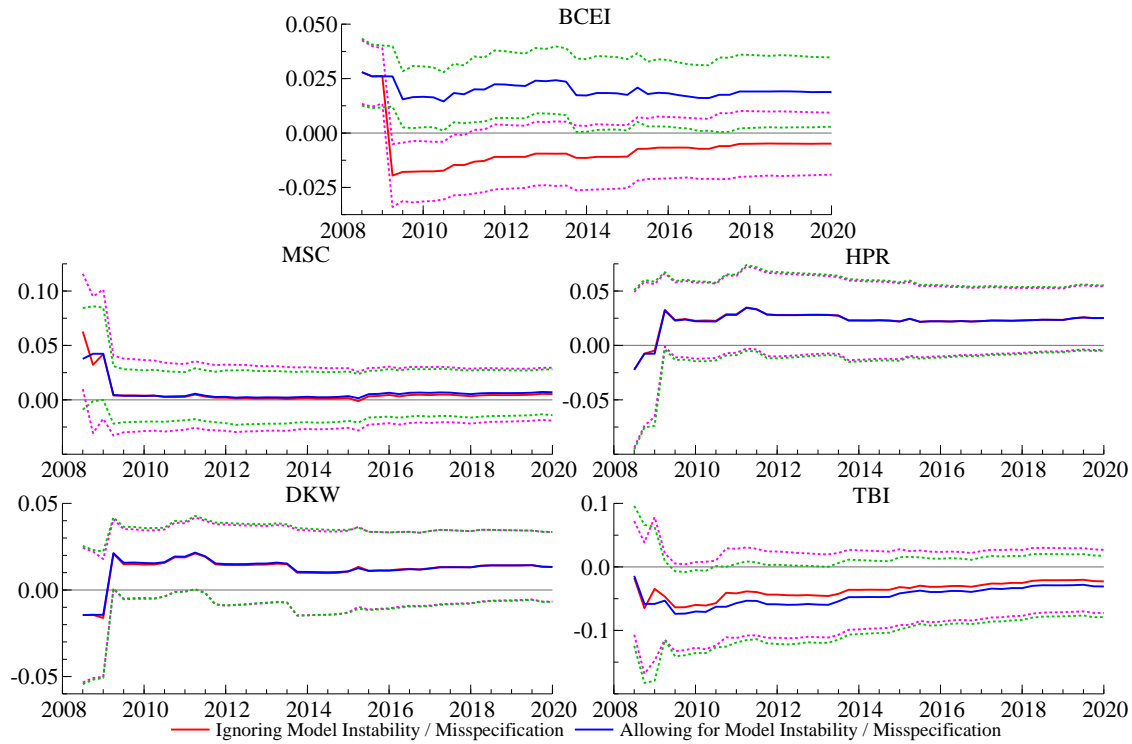
Figure A.1: Fluctuation Encompassing Tests

A.3 Additional Figures and Tables



Notes: DKW and TBI only contain information up through the end of the month prior to which they are released. The shaded area represents Q1.

Figure A.2: Illustrative Timeline of Release Dates for 2020 Q1



Notes: Estimates are based on an expanding estimation window. The dotted lines capture the 2 standard deviation uncertainty around the estimates. Note that the uncertainty is essentially unchanged when HAC estimates are used.

Figure A.3: Recursive Estimates of Rigidity

Table A.3: Forecast-Encompassing Coefficients and Probabilities Relative to BCEI

Horizon:	1	2	3	4	5	6	7	8	Joint
SPF	4.77 [0.298] {0.189}	2.54 [0.604] {0.392}	1.93 [0.732] {0.478}	4.33 [0.219] {0.111}	4.04 [0.321] {0.044}**	2.19 [0.647] {0.401}	0.61 [0.878] {0.808}	0.09 [0.667] {0.965}	0.77 [0.873] {0.754}
LIV	10.00 [0.023]** {0.012}**	2.03 [0.776] {0.575}	1.47 [0.896] {0.681}	3.00 [0.532] {0.349}	2.05 [0.696] {0.447}	-3.30 [0.122] {0.233}	-3.58 [0.144] {0.252}	-3.12 [0.060]* {0.152}	-3.09 [0.001]** {0.001}***
ARU	0.82 [0.969] {0.862}	-2.33 [0.367] {0.527}	-1.32 [0.524] {0.716}	3.30 [0.493] {0.327}	5.76 [0.159] {0.090}*	1.56 [0.876] {0.663}	-1.66 [0.499] {0.673}	-2.29 [0.286] {0.457}	-1.77 [0.120] {0.154}

Notes: Horizon is number of quarters-ahead that are being forecast. The values in each block are 1): the estimated coefficients from equation (3) with a dummy variable for 2008 Q4, 2): The p-value associated with the null-hypothesis that the coefficient is equal to unity in the square brackets; and 3): The p-value associated with the null hypothesis that the coefficient is equal to zero in the squigly brackets. All tests use HAC estimates from Andrews (1991). * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

Table A.4: Additional Joint Forecast-Encompassing Tests

	MSC		HPR		DKW		TBI	
	(1): Base	(2): (1) ∇U	(3): Base	(4): (3) ∇U	(5): Base	(6): (5) ∇U	(7): Base	(8): (7) ∇U
BCEI	-2.31 [[0.249]] {0.417}	-2.31 [[0.653]] {0.561}	1.67 [[0.310]] {0.008}***	2.12 [[0.446]] {0.063}*	1.45 [[0.904]] {0.053}*	1.46 [[0.980]] {0.932}	-0.49 [[0.338]] {0.436}	-0.49 [[0.019]]** {0.048}**
SPF	-3.84 [[0.065]]* {0.114}	-3.84 [[0.167]] {0.212}	1.97 [[0.195]] {0.014}**	2.71 [[0.378]] {0.075}*	1.76 [[0.695]] {0.080}*	1.78 [[0.997]] {0.992}	-0.68 [[0.126]] {0.287}	-0.68 [[0.003]]*** {0.015}**
LIV	-3.13 [[0.221]] {0.473}	-3.12 [[0.741]] {0.830}	2.05 [[0.132]] {0.005}***	2.66 [[0.227]] {0.034}**	2.00 [[0.720]] {0.127}	2.03 [[0.976]] {0.988}	-0.51 [[0.234]] {0.455}	-0.50 [[0.010]]** {0.053}*
ARU	-2.67 [[0.166]] {0.351}	-2.80 [[0.459]] {0.338}	1.91 [[0.117]] {0.002}***	2.34 [[0.362]] {0.074}*	1.66 [[0.858]] {0.093}*	1.67 [[0.994]] {0.934}	-0.36 [[0.492]] {0.597}	-0.37 [[0.042]]* {0.129}

Notes: The values are the estimated coefficients from equation (3) with a dummy variable for 2008Q4. The estimates and tests follow as a special case from Hungnes (2018). ∇U represents the unemployment gap constructed using using quarterly unemployment and real-time estimates of the NAIRU from CBO and the Federal Reserve Board. Measures along the rows represent the base forecasts. The p-value associated with the null-hypothesis that the coefficient is equal to unity is in the square brackets. The p-value associated with the null hypothesis that the coefficient is equal to zero is in the squigly brackets. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

Table A.5: Estimates of γ_m for Alternative Measures of Professional Forecaster Expectations

	BCEI	SPF	LIV	ARU
No Model Instability:	-0.005 (0.007)	0.025*** (0.009)	0.002 (0.005)	0.011** (0.005)
Model Instability / Misspecification:	0.019** (0.008)	0.022** (0.009)	0.001 (0.003)	0.011** (0.005)

Notes: All equations are estimated from 2003Q2 - 2019Q4 (67 observations) with a constant. We allow for model instability by selecting over and retaining potential outliers and shifts using Autometrics with Impulse Indicator Saturation (IIS) and Differenced-IIS (DIIS). The target gauge is 0.1% so that under the null hypothesis we expect 0.2 irrelevant indicators to be retained on average. In all equations at least one outlier for 2008 Q4 is retained while at most there are five: see Supplemental Material. The estimated standard errors are in parentheses. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

Table A.6: Dates of Detected Outliers and Differenced Outliers

Date	BCEI	SPF	LIV	MSC	ARU	HPR	DKW	TBI
2006 Q4		DI						DI
2008 Q2				DI				
2008 Q4	I	I	I	I	I	I	I	I
2009 Q1	I							
2010 Q3				DI				DI
2010 Q4			I					
2011 Q4			I					
2013 Q4			I					
2014 Q4			I					

Notes: I = Impulse and DI = Differenced Impulse.

Table A.7: Selected Variables by Information Differential and Target Gauge

Target Gauge:	Base		Dummy: 2008Q4	
	0.1%	0.5%	0.1%	0.5%
∇f_{MSC} :				
Housing Permits (west)			x	x
PPI: Crude Materials	x	x		
Real Personal Income	x	x		
Real Personal Income (excl. transfers)	x	x		
Reserves of Depository Institutions			x	x
∇f_{HPR} :				
Real Money Stock (M2)	x	x	x	x
∇f_{DKW} :				
S&P 500 Index: Composite	x	x	x	x
S&P 500 Index: Industrials	x	x	x	x
∇f_{TBI} :				
New Orders for Consumer Goods		x		
Real Personal Income		x		
Real Personal Income (excl. transfers)		x		
Reserves of Depository Institutions	x			
S&P 500 Index: Industrials		x		

Note: All columns include Oil prices forced into the equation.