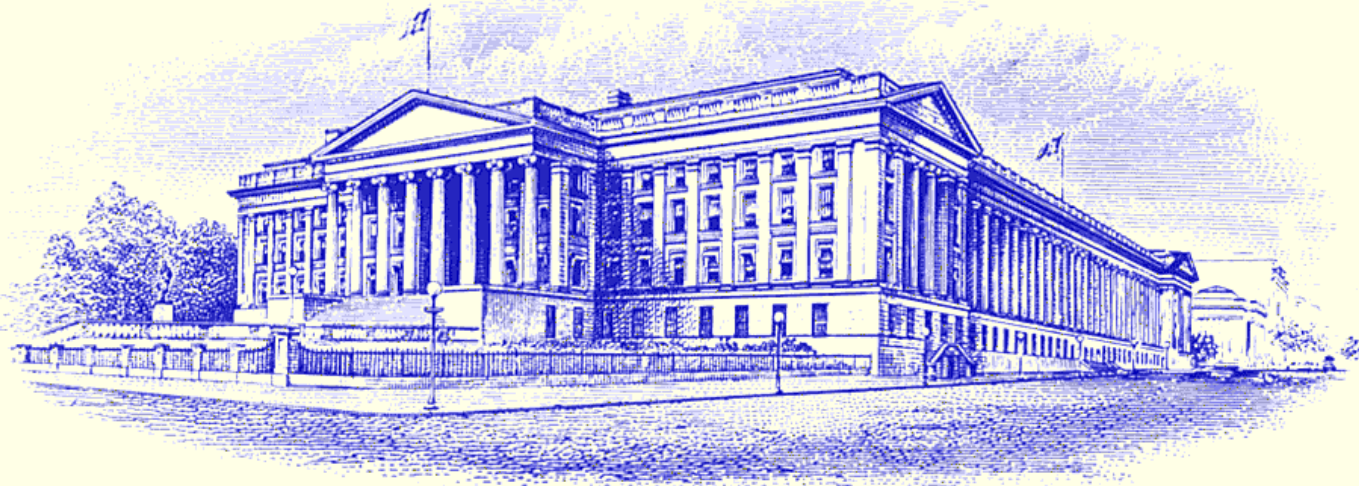


TREASURY YIELD CURVES AND DISCOUNT RATES



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Introduction

- This presentation discusses three yield curves that provide discount rates for calculating present values of future cash flows:
- The first curve is the Treasury Nominal Coupon-Issue (TNC) Yield Curve, which pertains to Treasury nominal coupon issues.
- The second is the Treasury Real Coupon-Issue (TRC) Yield Curve for Treasury Inflation-Protected Securities (also known as TIPS).
- And the third is the High Quality Market (HQM) Corporate Bond Yield Curve, which pertains to U.S. high quality corporate bonds.
- The presentation summarizes information about the three yield curves with emphasis on the Treasury curves. More information can be found in the references in the last slide.

Yield Curve Definition

- A *yield curve* provides information about a sector of the bond market at a point in time. The information includes yields on different types of bonds in this sector at various maturities.
- For the TNC and TRC curves, the sectors are Treasury nominal coupon issues and TIPS, respectively. For the HQM curve, the sector is U.S. high quality corporate bonds.

The Par Yield Curve

- One set of information provided by yield curve analysis is the par yield curve.
- The par yield curve shows for each maturity the yield on a security of that maturity that is selling at par (price excluding accrued interest equals 100).
- The par yield curve provides a picture of the respective market sector for securities with coupons and is used for market analysis. Time series of par yields show market movements over time.

The Spot Yield Curve

- However, for the purpose of discounting future cash flows, another set of information provided by yield curve analysis is more useful, and that is the spot yield curve.
- The spot yield curve shows for each maturity the yield on a security without coupons that provides a single payment at that maturity. Such a security can be called a zero coupon bond. The yields are called spot rates.
- All the yield curves discussed here are estimated from coupon securities; there are no actual zero coupon securities in the estimation. Therefore, the spot rates are calculated so that they are consistent with the yields on the coupon securities, and they can be obtained approximately in the market by a portfolio of coupon securities.

The Spot Yield Curve, continued

- The spot rates are the appropriate discount rates to be used for discounting future cash flows.
- Each future cash flow is discounted by the spot rate whose maturity is the same as the future point in time when the cash flow occurs to get the present value of the cash flow. The present values of a series of cash flows are then added to get the total present value of the series.
- Sometimes par yields are used to discount cash flows. This approach is flawed because par yields include the effects of coupon payments and do not match the cash flows in time.

The HQM Yield Curve

- The HQM corporate bond yield curve is produced as mandated by the Pension Protection Act of 2006 (PPA). This curve pertains to high quality corporate bonds, that is, bonds in the top three qualities AAA, AA, and A.
- The curve data are disseminated by IRS and by the Treasury Office of Economic Policy, and include spot rates and segment rates derived from the spot rates that are used by single-employer pension plans to discount future liabilities.
- To meet the requirements of the PPA, it was necessary to invent a new yield curve methodology at Treasury for the HQM curve. The methodology is described below.
- The HQM yield curve is available back through 1984.

The TNC Yield Curve

- The HQM methodology was subsequently applied to produce the TNC yield curve, which pertains to Treasury nominal coupon issues, both notes and bonds.
- The TNC curve includes both on-the-run issues (securities most recently issued of each maturity) and older off-the-run issues. However, in this presentation the focus is on spot rates from off-the-run issues, which can be derived for all maturities without on-the-run distortions and are more suitable for discount rates.
- The initial application of the TNC yield curve was to discount future liabilities of various federal agencies, such as pension and other postemployment liabilities, for the agencies' audited financial statements and for the annual Financial Report of the U.S. Government.
- The TNC curve is available back through 2003, and is being extended back another 25 years to the mid 1970s.

The TRC Yield Curve

- The HQM and TNC methodology was subsequently used to produce the TRC yield curve. The TRC yield curve is the real yield curve derived from Treasury Inflation-Protected Securities or TIPS, and includes both real notes and real bonds.
- The nominal TNC and real TRC yield curves can be combined to derive the Treasury Breakeven Inflation (TBI) Curve, where the breakeven inflation rate for any maturity is the inflation rate that equates nominal and real Treasury spot rates at that maturity in dollar terms. The TBI curve can be used to estimate inflationary expectations.
- The TRC yield curve is available back through 2003.

Yield Curve Data

- These yield curves each provide several sets of data, including spot rates, selected par yields, and forward rates. The TNC curve provides off-the-run and on-the-run data.
- The curves are calculated late in the day on each business day, and the results are disseminated each month. The spot rate data include monthly averages of daily spot rates, as well as end of month spot rates from the TNC and TRC curves.
- Spot rates are available for each maturity at half-year intervals starting at $\frac{1}{2}$ year up through 100 years, for a total of 200 maturities. Rates beyond 30 years maturity are projected as described below.

Yield Curve Data, continued

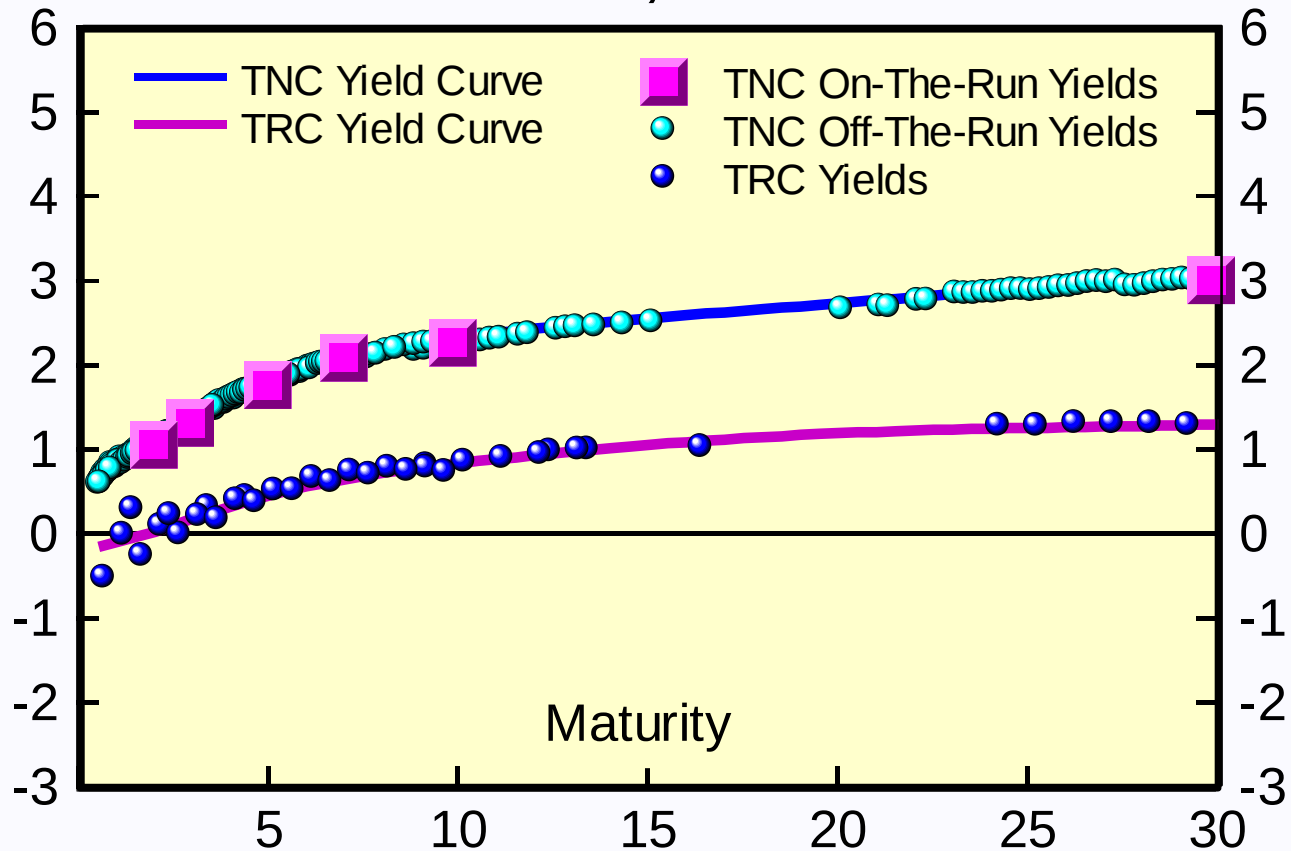
- All spot rates are semiannually compounded, following market convention. However, some applications for discounting may choose to ignore the semiannual compounding and apply the rates as if they were annual.
- Off-the-run TNC and TRC spot rates can be interpreted as the risk-free social rates of time preference. This means that social preference is indifferent between future cash flows and their present values discounted by the TNC spot rates in the case of nominal flows or TRC spot rates for flows in real terms.

Yield Curves for December 31, 2015

- The next chart contains the par yield curve for this day.
- There are: 278 Treasury nominal coupon issues in the TNC dataset for this day including 6 on-the-run issues; 37 TIPS in the TRC dataset; and 1,600 AAA, AA, or A rated securities plus 9 AA commercial paper rates in the HQM yield curve dataset.
- For this day, all yields are close to the respective curves.
- TIPS yields are negative at the earliest maturities.

TNC and TRC PAR YIELD CURVES

12/31/2015, Percent



Yield Curves for December 31, 2015, continued

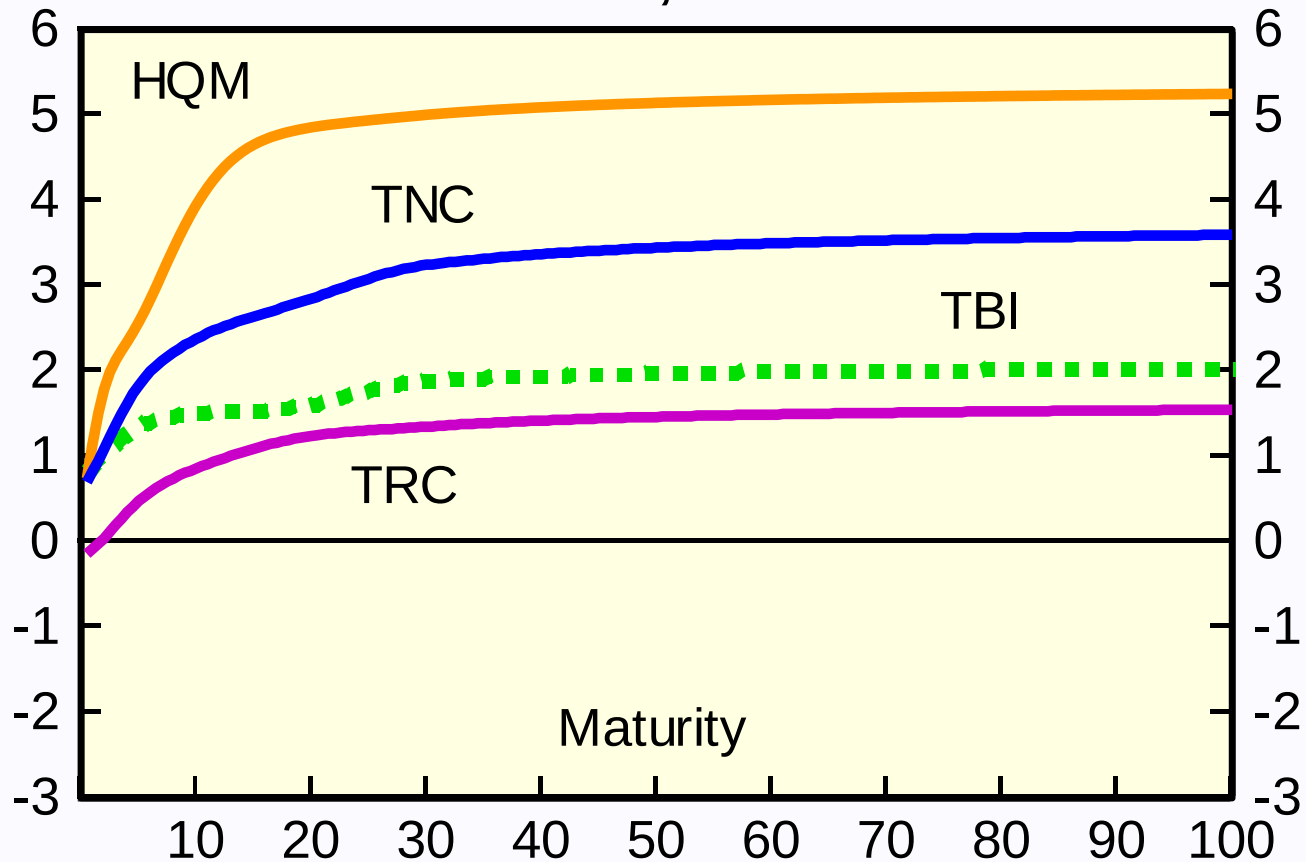
- Slide 16 contains TNC, TRC, and HQM spot yield curves for this day plus the TBI breakeven curve which is derived from the TNC and TRC spot curves. All curves are projected out to 100 years maturity.
- The HQM spot rate is 5.00 percent at 30 years maturity, compared with 3.25 percent and 1.35 percent for the TNC and TRC spot rates, respectively. The TBI rate is 1.89 percent at 30 years maturity.
- The HQM rate exceeds the TNC rate largely because of corporate bond default risk, and the TNC rate exceeds the TRC rate with positive inflation.

Yield Curves for December 31, 2015, continued

- The spot yield curves rise gradually throughout the projection range from 30 years maturity through 100 years maturity, which is typical. At 100 years maturity, the HQM spot rate is 5.25 percent, the TNC rate is 3.60 percent, the TRC rate is 1.55 percent, and the TBI rate is 2.04 percent.
- The real TRC spot yield curve is negative at the earliest maturities. Discounting with negative spot rates actually increases present values.

SPOT YIELD CURVES

12/31/2015, Percent



Methodology Features

The methodology used for the yield curves contains features and capabilities that do not appear in other yield curve approaches:

- The methodology makes use of established bond market characteristics to generate a stable yield curve that captures market movements.
- The methodology projects yields beyond 30 years.
- The methodology combines regression variables with the yield curve.

The following slides discuss details of these features.

Methodology Features, continued

- The requirements of the PPA made necessary the invention of this methodology, which is new and more extensive and more powerful.
- Because different yield curve methodologies can produce significantly different results which affect discounted cash flows and everything else for which yield curves are used, it was essential that Treasury develop and implement the methodology that most accurately captures market behavior.

Methodology Features, continued

- In particular, the mathematical form of the yield curve must be selected in order to compute the curve.
- Other yield curve approaches typically use mathematical forms that don't have any foundation in bond market characteristics. Consequently, differences among yield curves given by other approaches with different mathematics are arbitrary, in that there is no way to choose among them.
- In contrast, the methodology used here is grounded in established bond market characteristics, which generate the mathematical form of the yield curve and ensure that the form is not arbitrary.

The Forward Rate

- The concept of the forward rate is useful for constructing a mathematical form for the yield curve that accords with established market characteristics.
- The forward rate is easy to define: for each maturity, consider entering into a contract to invest some money at the time of that maturity for a small amount of time beyond that maturity. The forward rate at that maturity is the future interest rate on this investment.
- The forward rate is higher at a given maturity when investors who are trading at that maturity are less eager to lend based on their assessment of uncertainty and their expectations and purposes, while borrowers are more eager to borrow based on their perceptions. The forward rate summarizes market views for each maturity in a single number.

Maturity Ranges

- Moreover, trading in securities tends to divide into maturity ranges, such that the trading activity in each range on average reflects similar purposes, similar views of risk, and similar expectations about securities in that range.
- Because market views can be considered similar for securities in the same range, the forward rates in each maturity range can be assumed to be related to each other in a simple fashion.
- Consequently, this methodology models the forward rates in each maturity range as a smooth (cubic) function, and joins the functions together smoothly across ranges (as a cubic spline).

Maturity Ranges, continued

- This methodology at the present time uses five maturity ranges, delineated by the maturity points 0, 1.5, 3, 7, 15, and 30 years maturity. These points provide separate ranges for the critical maturities of 2 years, 5 years, the benchmark 10 years, and 30 years.
- The choice of fixed maturity ranges increases significantly the stability of the yield curve estimates over time.
- In addition, the methodology is statistically straightforward to estimate. In contrast, certain other yield curve approaches generate statistical models that are ill-conditioned and unstable.

The Long-Term Forward Rate

- The methodology must include the capability of projecting forward rates beyond 30 years maturity so as to obtain yields out through 100 years maturity. This is necessary because cash flows to be discounted frequently extend beyond 30 years.
- First of all, the long-term forward rate at and beyond 30 years maturity is set to a constant, because there are not enough data to estimate accurately movements in the forward rate beyond 30 years maturity.

The Long-Term Forward Rate, continued

- Next, this methodology postulates that the long-term forward rate is determined by the same factors that affect forward rates in the farthest 15- to 30-year maturity range, since that range is sufficiently distant in time to reveal underlying long-term attitudes toward risk and return.
- Therefore, the constant long-term forward rate from 30 years maturity up through 100 years maturity is taken to be the average forward rate in the 15- to 30-year maturity range.

Methodology: Projections

- Using the long-term forward rate, spot rates can be projected beyond 30 years maturity out to 100 years maturity. The projected spot rates provide discount rates for long-dated cash flows.
- Moreover, the method of construction of the long-term forward rate ensures that yields around 30 years maturity are consistent with yields before 30 years maturity and with long-term investment returns available in the market.
- Other yield curve approaches generally stop at 30 years maturity and contain no provision for projection and no mechanism to ensure that yields around 30 years maturity are consistent with earlier yields. Building the projection methodology into the yield curve solves these problems.

Methodology: Regression Variables

- This methodology also has the special capability of combining regression variables with the yield curve. This capability is not available in other approaches. The regression variables adjust for various features of the market and particular attributes of individual securities.
- The TNC and TRC yield curves use regression to measure the hump in yields that is sometimes seen around 20 years maturity.
- The TNC curve also uses regression to measure price effects of on-the-run securities. As opposed to other approaches, the TNC yield curve includes both on- and off-the-run securities in the same yield curve and measures on-the-run effects.

Data

- The TNC yield curve represents all nominal coupon issues and the TRC yield curve represents all TIPS. The dataset for each business day is derived from securities that are priced on that day. Bid prices are used.
- The HQM yield curve represents all high quality corporate bonds that have similar features to Treasury coupon issues.
- The dataset for each day excludes securities with fewer than two coupon payments remaining.

Estimation

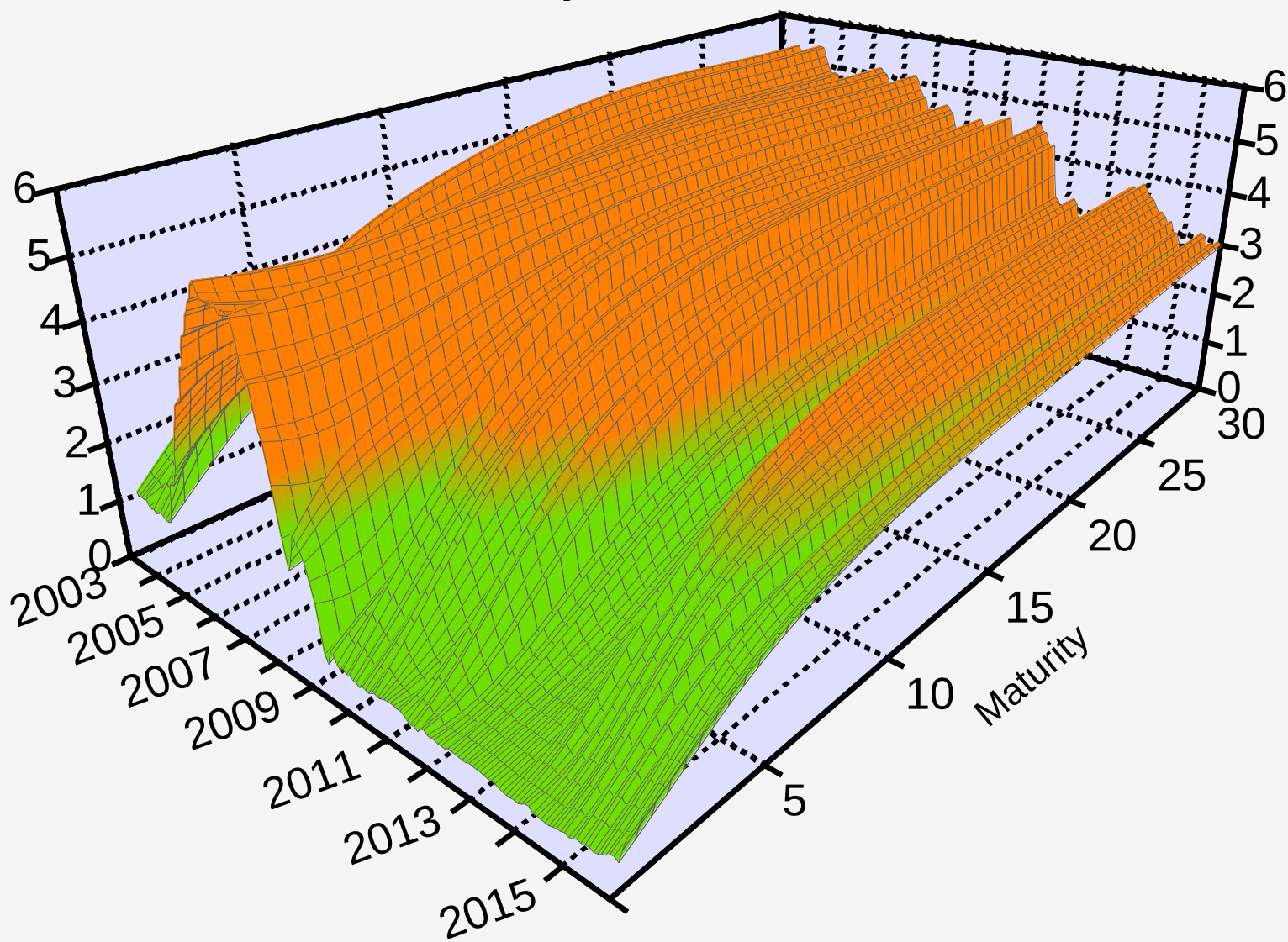
- The forward rates in the maturity ranges are chosen to be those rates that give the best statistical fit (by least squares) to all the prices in the dataset for that day. The par and spot yields are derived from the estimated forward rates.
- Regression terms are simultaneously estimated with the forward rates.
- Before estimation, the data are weighted by the (square root of the) inverse of duration.

Results for the TNC Yield Curve

- The following charts show results for the TNC yield curve over the period January 2003 through December 2015, for a total of 3,247 business days and 156 months.
- The first chart in slide 30 shows the monthly average par yield curve over the entire period.
- The chart shows that the par yield curve flattened out before the 2007-2009 recession, and that yield spreads were large in the financial crisis of 2009 forward. After that, spreads decreased a bit.

TNC PAR YIELD CURVE

Monthly, Percent

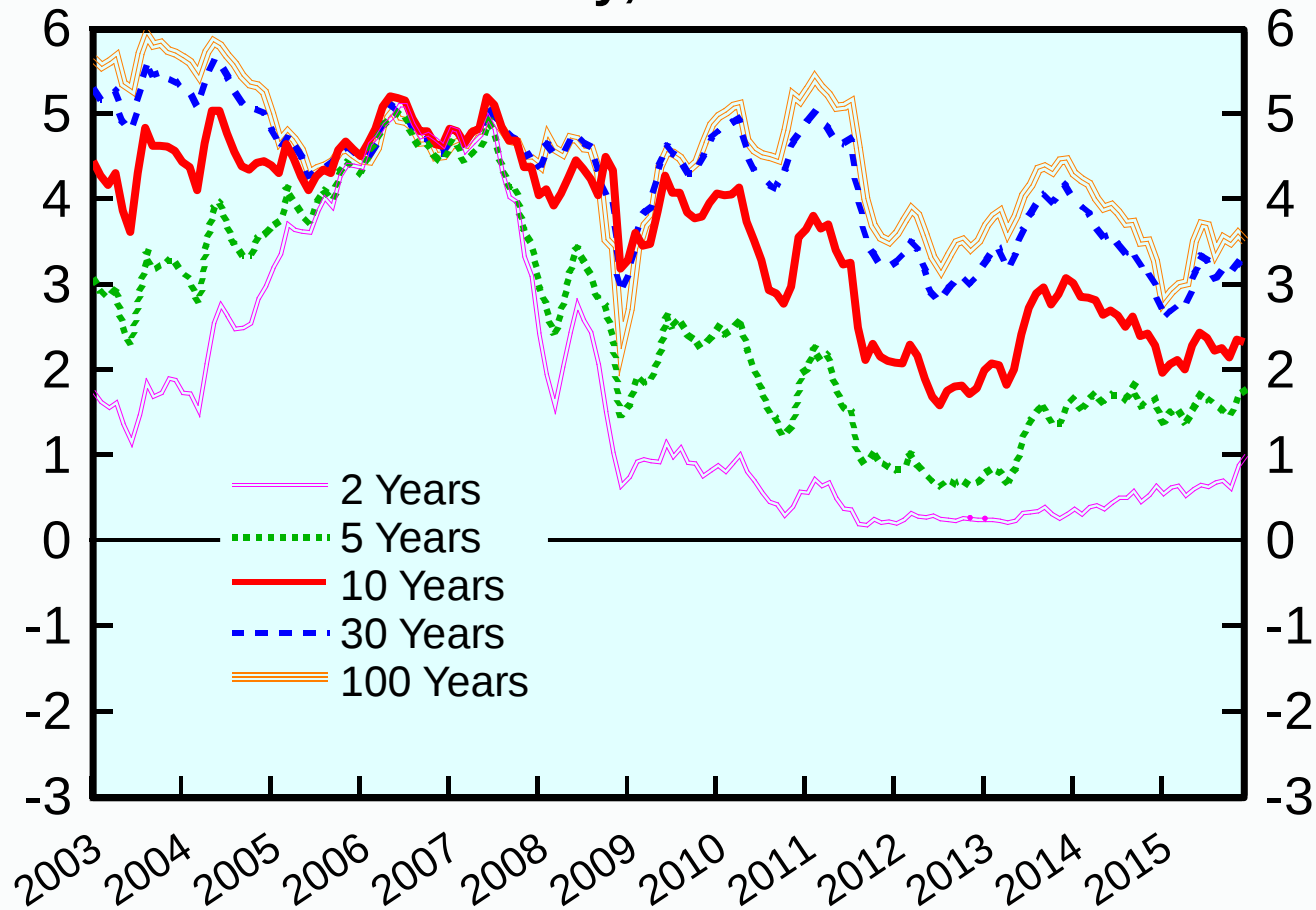


Results for the TNC Yield Curve, continued

- The second chart in slide 32 shows monthly average spot rates over the period at maturities of 2 years, 5 years, 10 years, 30 years, and 100 years.
- Again there is a significant bunching of the spot rates before the 2007-2009 recession.

TNC SPOT RATE AT SELECTED MATURITIES

Monthly, Percent

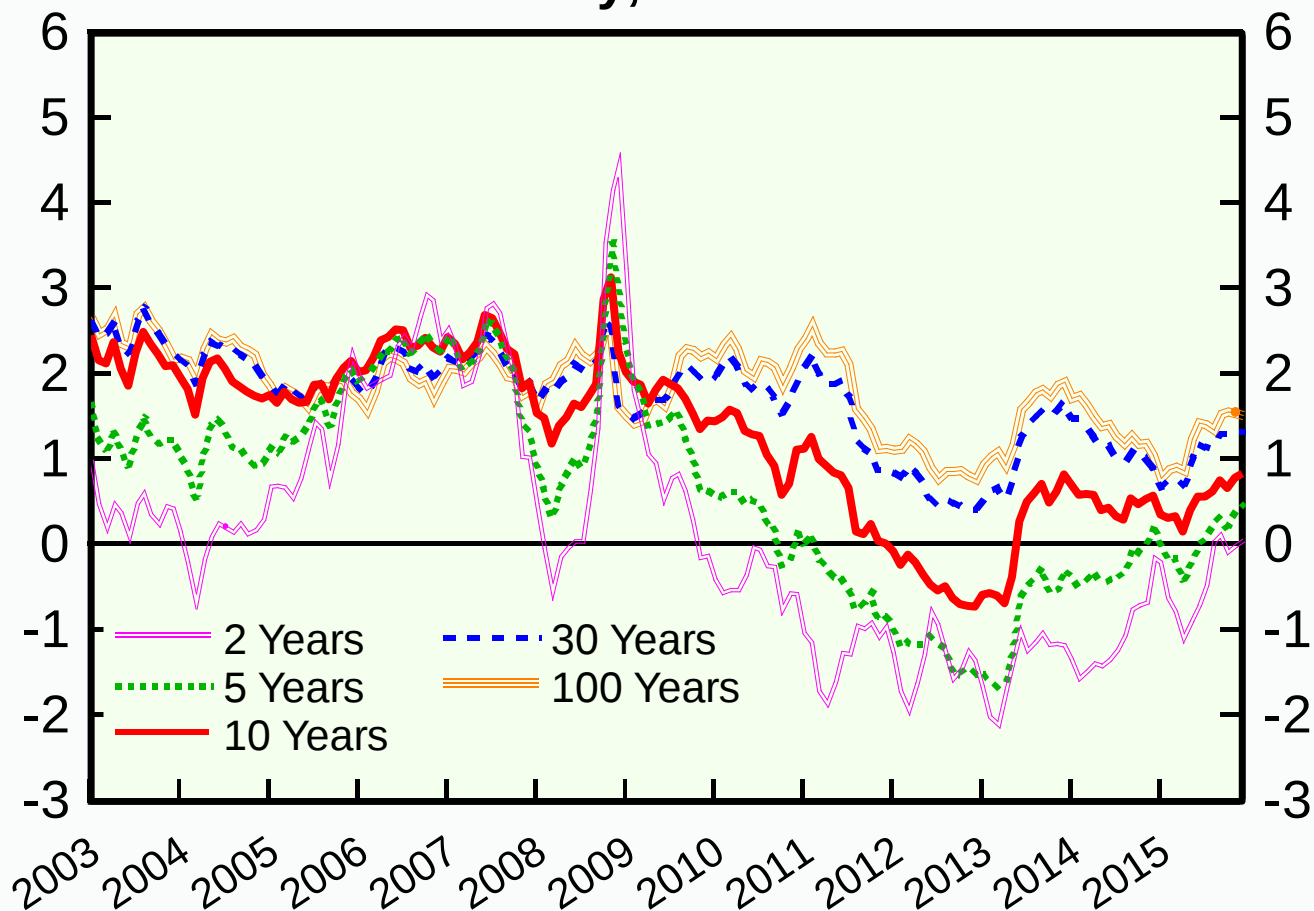


Results for the TRC Yield Curve

- The following chart in slide 34 shows the analogous spot rates for the TRC yield curve over the same period January 2003 through December 2015.
- Real TRC spot rates spiked at the end of 2008, due in part to reduced liquidity in the financial crisis.
- Similar to the nominal spot rates, the chart shows that the real rates bunched together before the 2007-2009 recession.
- Over the last few years, there have been episodes of negative spot rates at lower maturities.

TRC SPOT RATE AT SELECTED MATURITIES

Monthly, Percent



For More Information

- Data and documentation for the TNC and TRC yield curves are available on the Treasury Economic Policy website at:

<https://www.treasury.gov/resource-center/economic-policy/corp-bond-yield/Pages/TNC-YC.aspx>

- The HQM corporate bond yield curve for the PPA is published by the IRS each month. Data and documentation are also available on the Treasury Office of Economic Policy website at:

<https://www.treasury.gov/resource-center/economic-policy/corp-bond-yield/Pages/Corp-Yield-Bond-Curve-Papers.aspx>

- Previous applications of the methodology presented here to TIPS yield curves can be found in papers 0601 and 0501 in the Economic Policy Research Paper Series on the Office of Economic Policy website.