TBAC Charge: An Update on the TBAC Issuance Model – Incorporating TIPS

Please provide an update on efforts the Committee is making with regard to the development of issuance models, including any updated analysis or results and any revisions to or extensions of the modeling work that was presented in October 2017, particularly the incorporation of TIPS into the model. Comment on the degree to which the updated modeling efforts can be used by Treasury as one input to help to inform potential its decisions regarding nominal coupon and TIPS issuance.

Executive Summary

- This presentation extends the debt management model of Belton et al.[1] to assess the optimal mix and maturity structure of nominal and inflation linked debt. While model outputs should not be (and are not being) used prescriptively, they do provide a number of insights on how TIPS contribute to U.S. debt costs and risks.
- Under the model's structure, the debt service costs for TIPS issuance are generally lower than that of equivalent maturity nominal issuance because the risk premium required by investors as compensation for inflation risk exceeds what is required to compensate for liquidity risk.
  - Five year TIPS seem to offer the greatest cost advantage; however, ten year TIPS offer an attractive cost / risk trade off. Minimum issuance sizes in the thirty year point are useful in maintaining a long-dated benchmark.
  - Currently, the relative risk premium of nominal versus TIPS issuance appears lower than the longer run average. However, the dynamic optimal response function does not react sensitively to time varying inflation risk premium.
- TIPS issuance can reduce risk to the Treasury if kept to amounts that leave TIPS allocations as a moderate proportion of the debt stock.
  - TIPS principal accretion flows through interest expense and introduces significant debt service volatility in any given period, even though this accretion does not represent an actual funding need in that period.
  - Nevertheless, the negative correlation between CPI-U and U.S. primary deficits creates a significant diversification benefit for Treasury debt stock allocations containing TIPS. Assuming historical correlations hold going forward, total deficit volatility is reduced for TIPS allocations up to 13% of the debt stock.
- In summary, when accounting for their relative cost and capacity for risk reduction, the model suggests that the level of TIPS outstanding could range from just a few percent of the outstanding debt stock (for a debt manager less averse to risk) to as much as 14% of the outstanding debt stock (for a more risk averse debt manager). Currently, TIPS make up 9% of the debt stock.

[1]Belton, Dawsey, Greenlaw, Li, Ramaswamy, and Sack, "Optimizing the maturity structure of U.S. Treasury debt: A model-based framework", The Hutchings Center on Fiscal and Monetary Policy, Brookings Institution (October 10, 2018). https://www.brookings.edu/research/optimizing-the-maturity-structure-of-u-s-treasury-debt/

Review of Current Model and Extension

- The existing debt optimization model (which does not contemplate TIPS) contains:
  - A simulation module consisting of:
    - A macroeconomic model for the unemployment gap, core PCE inflation, the Fed Funds target rate, the rate of change of real GDP, the potential rate of change of real GDP, and the equilibrium real rate of interest
    - A model for the Treasury yield curve using expected Fed policy and term premium
    - A fiscal model for the primary budget deficit
  - A debt dynamics module that evolves current and future debt issuance
  - An optimization module that identifies low cost strategies given risk appetite and constraints and can generate:
    - Static optimizations (issuance fractions never change)
    - Dynamic optimizations (issuance fractions depend on macro variables)

- In order for the original model to be extended, it needed to be re-implemented, and outputs cross-referenced with the original.

- In order to include TIPS, the re-implemented model had to be extended to include:
  - Headline CPI in the macroeconomic model
  - A model for the TIPS yield curve consistent with the existing model implementation, which involves a decomposition of term premium into inflation, real rate, and liquidity components
  - The addition of TIPS to the debt dynamics module
  - The inclusion of TIPS in the optimization module (both static and dynamic)

- By including TIPS, we aim to assess the optimal issuance allocation across nominal and inflation linked securities as well as optimal issuance points for each.

Term Premium Decomposition

We decompose TIPS breakevens by extending the model of AACM[1] to include 30Y yield curves[2]

- TIPS investors need to be paid a risk premium for real rate risk (RRP), while investors in nominal Treasuries must be paid an extra risk premium for taking inflation risk (IRP). The sum of the IRP + RRP is the nominal term premium (TP), which was modeled by Belton et al.

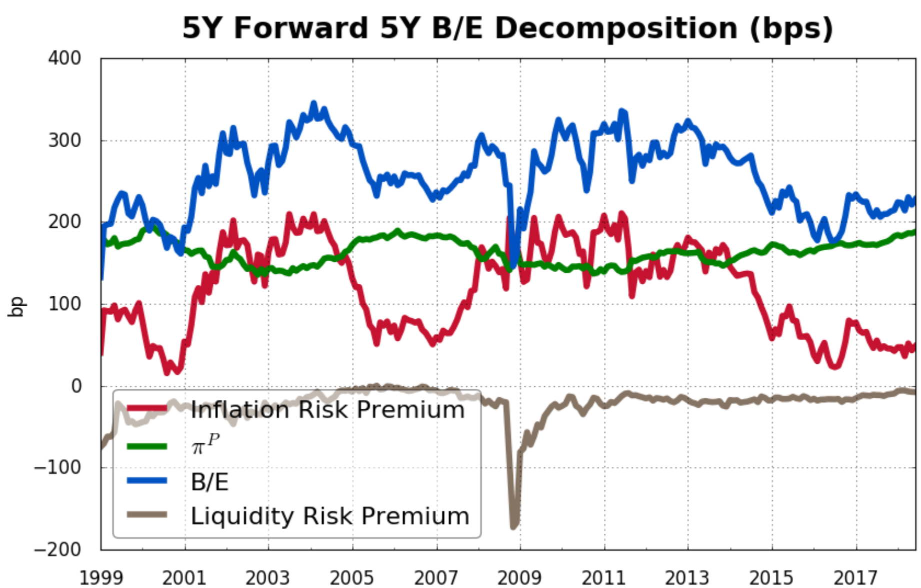
$$\text{Nominal yield} = \text{expected inflation} + \text{expected real yield} + \text{IRP} + \text{RRP}$$

- In addition, a liquidity risk premium (LRP) for TIPS is necessary in order to provide a sensible yield decomposition of nominal and inflation-linked Treasuries into expected inflation, expected real yield, inflation risk premium, and real rate risk premium.

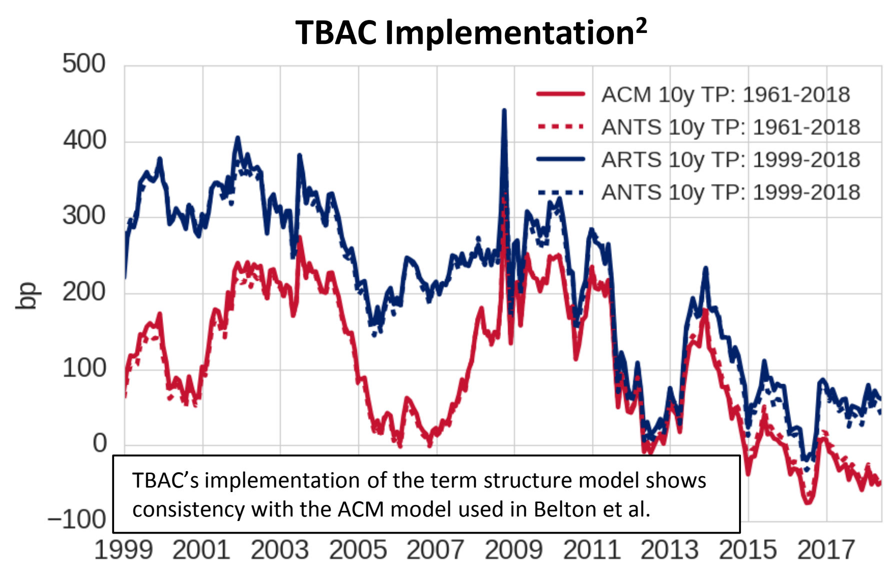
$$\text{TIPS yield} = \text{expected real yield} + \text{RRP} + \text{LRP}$$

- Market-implied breakeven inflation, which is the difference between equal maturity Treasury and TIPS yields, leads to counter-intuitive results during periods of low market liquidity unless it is adjusted for LRP.

$$\text{Breakeven Inflation} = \text{Nominal yield} - \text{TIPS yield} = \text{expected inflation} + \text{IRP} - \text{LRP}$$



Source: TBAC ARTS



Source: TBAC Rates Module, Federal Reserve Bank of New York

[1] Abrahams, Michael, Adrian, Tobias, Crump, Richard K., and Moench Emanuel, "Decomposing Real and Nominal Yield Curves", Federal Reserve Bank of New York Stat Reports (February 2015). https://www.newyorkfed.org/medialibrary/media/research/staff_reports/sr570.pdf

[2] In what follows, we refer to TBAC's implementation of the AACM model as ARTS (Affine Real Term Structure) when including TIPS and ANTS (Affine Nominal Term Structure)when using only nominal Treasuries.

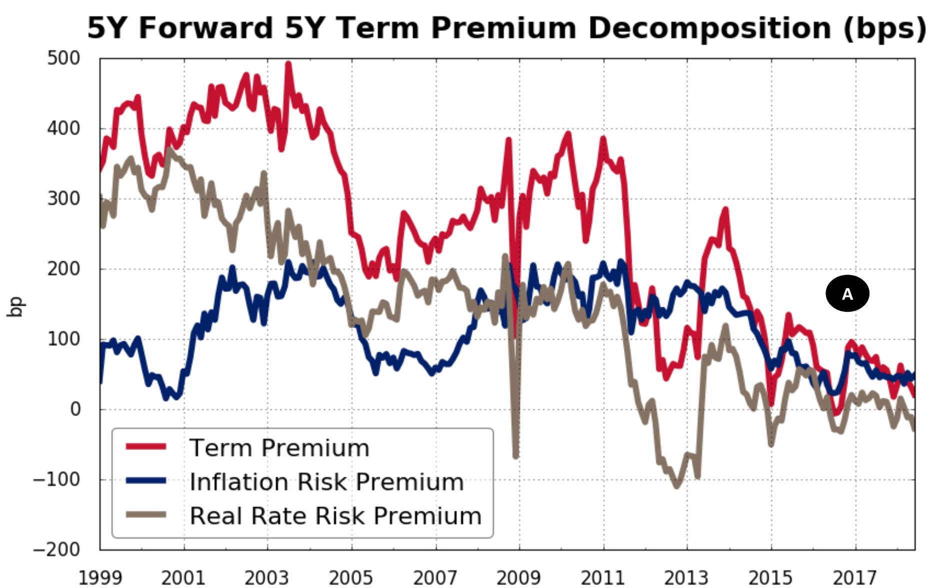
Inflation and Real Rate Risk Premiums

Inflation risk premium is modeled to vary with monetary policy; steady state behavior is based on recent averages

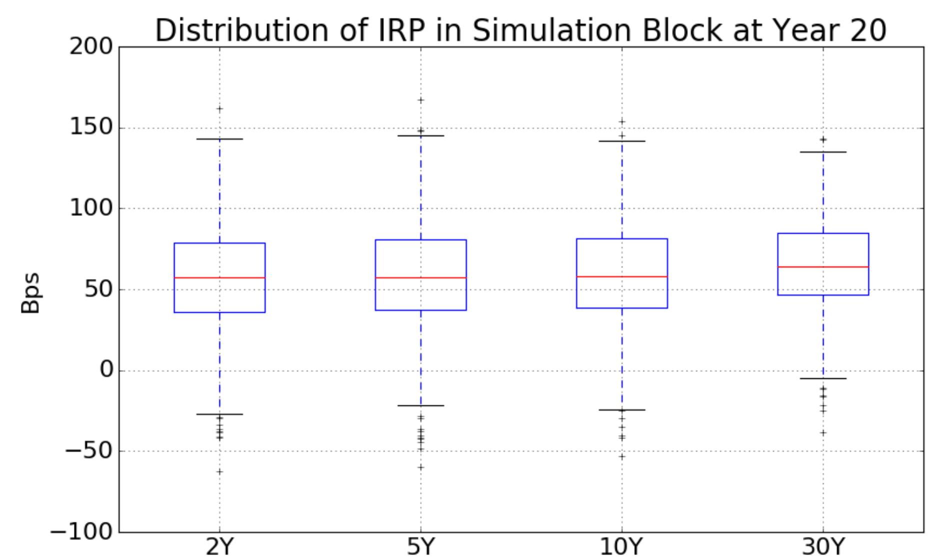
- The term premium is the sum of inflation risk premium and real rate risk premium.

A Output of the ARTS model shows more of the variation in TP can be explained with RRP. IRP is more steady.

- In our simulation module, we model IRP directly, and derive RRP as the difference between TP and IRP.
- The model for TP in our simulation module remains the same as in Belton et al.
- In the simulation module, we model expected real rates $r_{t,\tau}^{P}$ as the difference between expected nominal rates and expected inflation.
- We then write 5y and 10y IRP as affine functions of $r_{t,\tau}^{P}$ - r_t^*.
- Slope coefficients are estimated from regressions of ARTS model outputs onto the above variables, and the constant term is chosen to set the long-term expected level of IRP.
- IRP for other maturities is obtained from IRP5 and IRP10 using historical regression of ARTS model outputs.



Source: TBAC ARTS

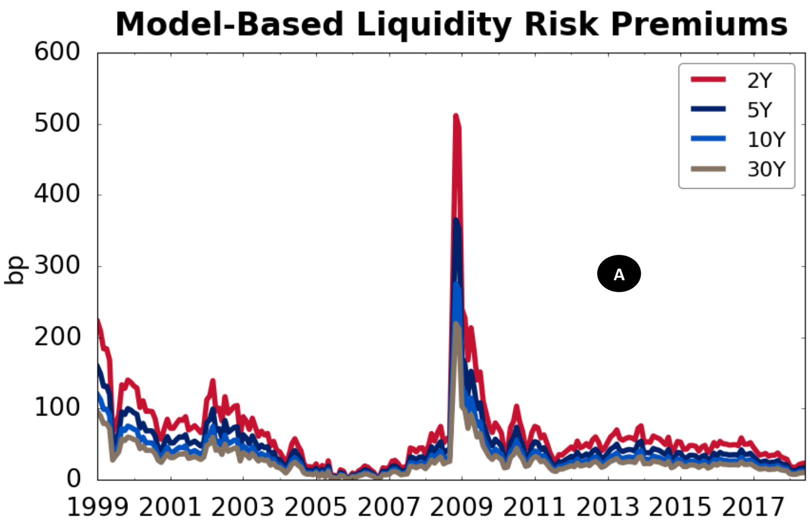


Source: TBAC Simulation Module

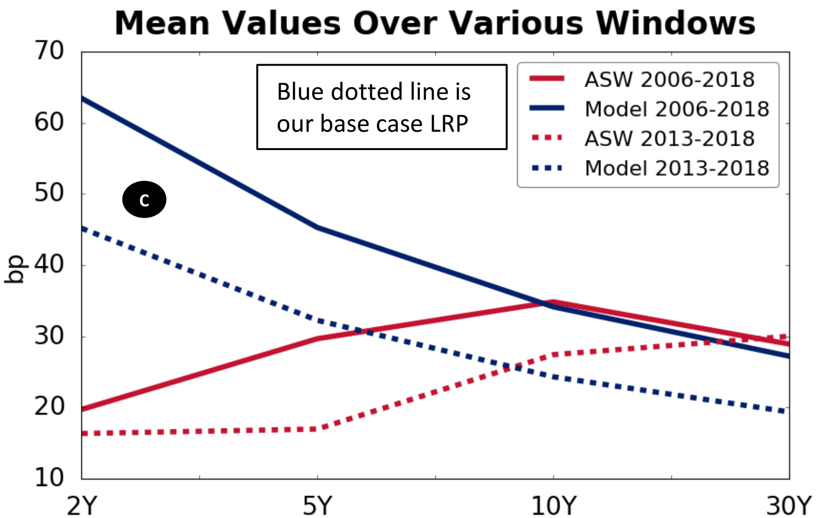
Liquidity Risk Premium

Model based estimates and market observables can be used to approximate TIPS liquidity risk premiums

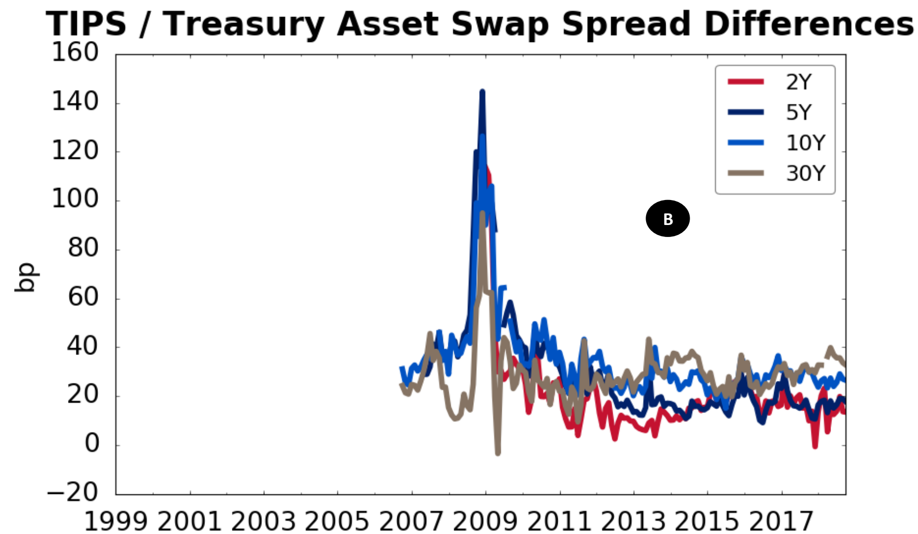
- **A** The ARTS model uses TIPS yield curve fit errors and trade volume data to generate liquidity risk premiums for TIPS over the entire calibration window (1999-present).
- **B** In the period for which there exists data on asset swap levels, the model based estimates are broadly similar (and in particular pick up the massive illiquidity during the financial crisis), but there are differences.
- **C** Comparing the term structures, we see that the model tends to generate larger liquidity premiums for shorter dated TIPS than is observed in the asset swap market.
- We use model liquidity premiums as our base case for TIPS but also show results using asset swap levels instead (the differences are marginal).



Source: TBAC ARTS



Source: Barclays, TBAC ARTS



Source: Barclays

Single Security Issuance Results for TIPS

Results show average debt service cost in year 20 vs two different measures of variance across the path population

- The scatterplots below introduce metrics for the cost vs. risk visualization and optimization we will be using throughout this work.

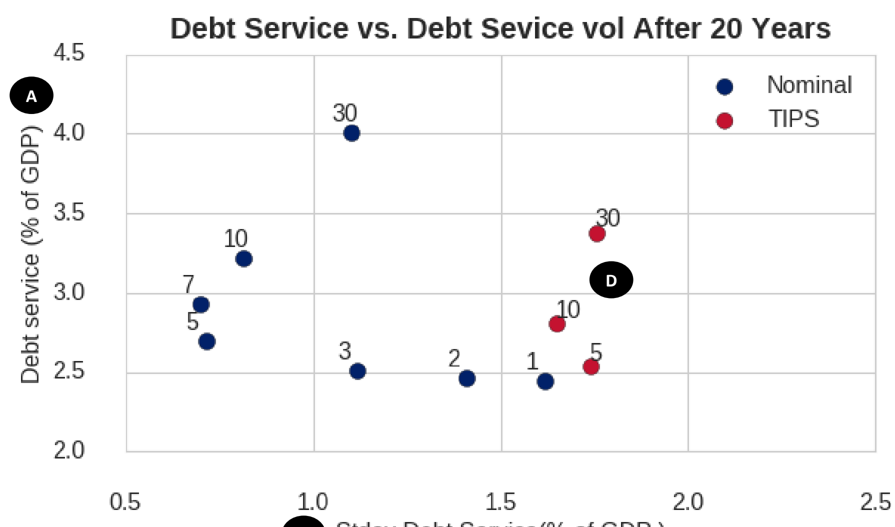
- In these simulations, cash needs are met every quarter entirely by issuance of a single security whose stock would, in the steady-state, finance the entire debt.

A The cost we look to minimize, on the vertical axis, is the average debt service cost (across all 2000 paths) at year 20 of our simulation.

B The risk on the right graph is the standard deviation (across all 2000 paths) of the total deficit (primary deficit + funding cost), which we continue to use throughout what follows.

C However, on the left we also show standard deviation (across all 2000 paths) of the debt service cost, as a touchpoint back to Belton et al. The blue dots show results for nominals, in close agreement with previous work.

D We are adding the red dots (TIPS), which for like tenor, are more volatile (shifted right), but also have lower cost (shifted lower), as holders of nominals must be compensated for the inflation risk premium.



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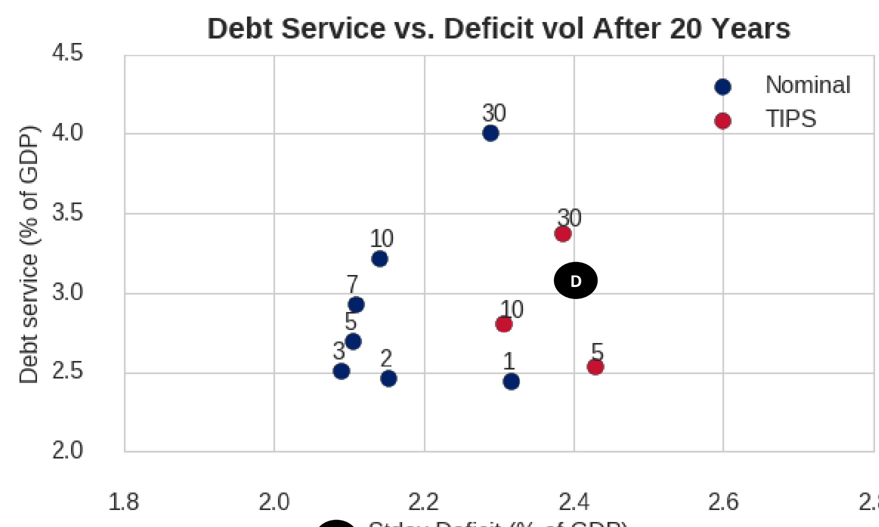

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Single Security Issuance Results for TIPS

Results show additional cost and risk summary statistics for single-security strategies

	1yN	2yN	3yN	5yN	7yN	10yN	30yN	2yT	5yT	10yT	30yT
Average issuance rate	2.99	2.96	2.99	3.10	3.25	3.44	4.01	1.18	1.04	1.21	1.63
Average debt service / GDP	2.44	2.46	2.51	2.69	2.93	3.21	4.00	2.71	2.53	2.80	3.37
Standard deviation debt service/GDP	1.62	1.41	1.12	0.72	0.70	0.82	1.11	2.27	1.74	1.65	1.76
Standard deviation total deficit (%GDP)	2.32	2.15	2.09	2.11	2.11	2.14	2.29	2.67	2.43	2.31	2.38
Correlation funding cost, primary deficit (%GDP)	(0.14)	(0.18)	(0.11)	0.12	0.14	0.11	0.10	(0.19)	(0.11)	(0.16)	(0.15)

A

C C

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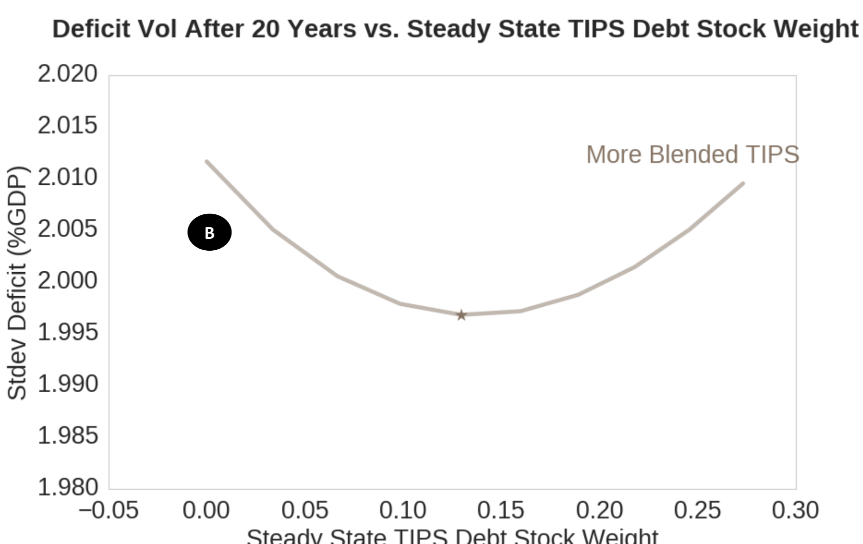
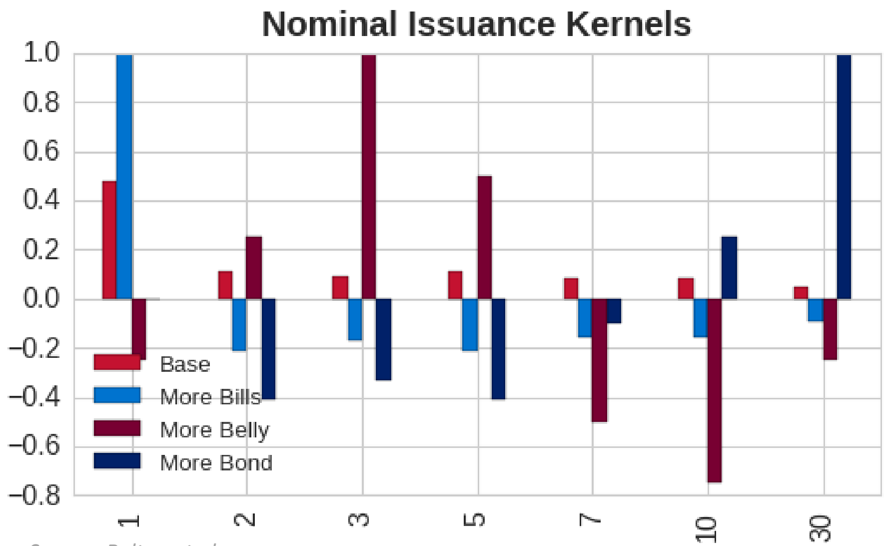


Source: TBAC

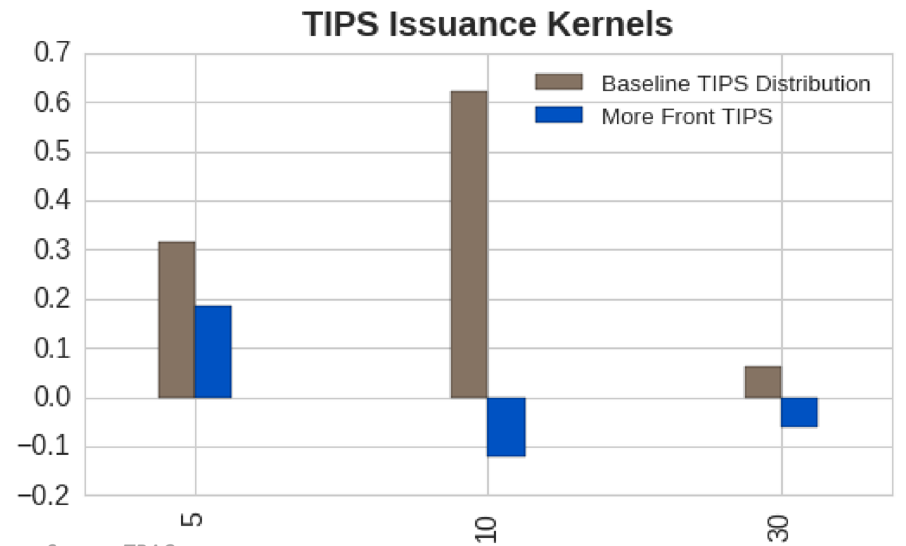
- **A** While TIPS are more volatile, they also have desirable correlation properties.
- **B** Since our primary risk metric is deficit volatility, where deficit = (funding cost + primary deficit), negative correlation between these two therefore lowers the volatility of the sum.
- **C** We see in the last row of the table that, for example, 5y TIPS show modest negative funding cost/ primary deficit correlation, while 5y nominals show small positive correlation.

Issuance Kernels for Nominals and TIPS

Reduce the issuance profile to a base-case which meets funding needs and several kernels



Source: Belton et al.



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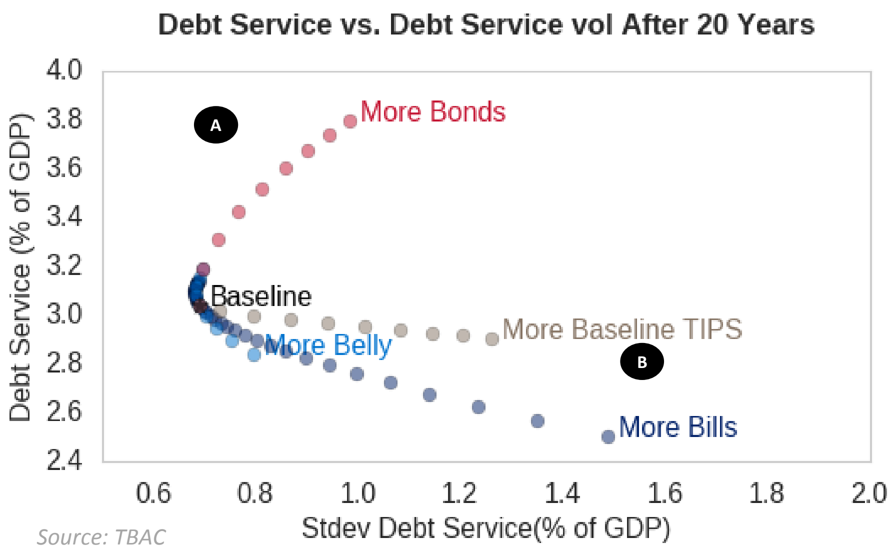
- One must be careful in specifying issuance kernels in terms of issuance, in order to take into account the implications for the steady-state debt distribution (see Appendix slides 21 and 22 for additional detail).

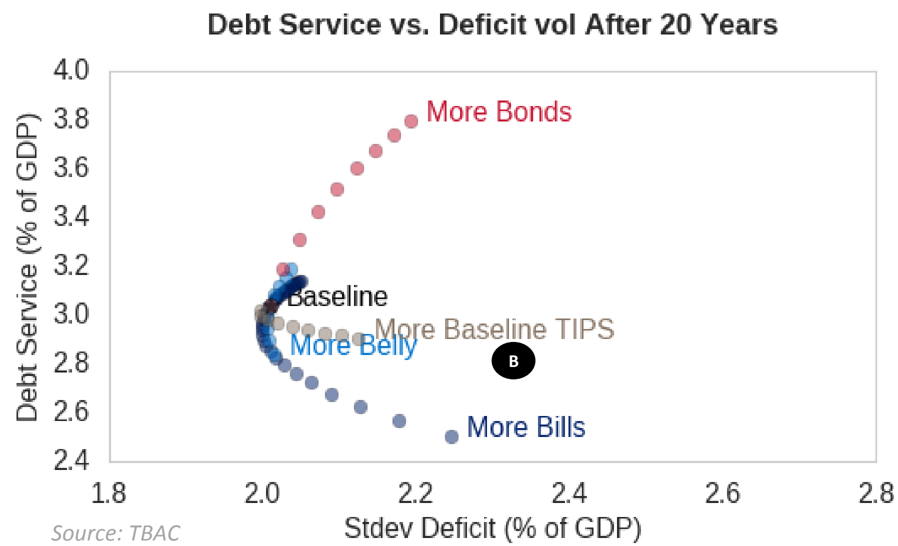
- Long-term issuance will pile up. For example, the baseline issuance kernel in Belton et al. would leave a large stock of original-issue 30y bonds after 20 years of issuance (5% of the quarterly issuance leads to 34% of the debt stock).

- The baseline TIPS kernel above is intended to replicate the current maturity distribution of TIPS after 20 years.

Issuance Kernels for Nominals and TIPS

Frontier plots allow us to see the risk / cost contribution of each kernel





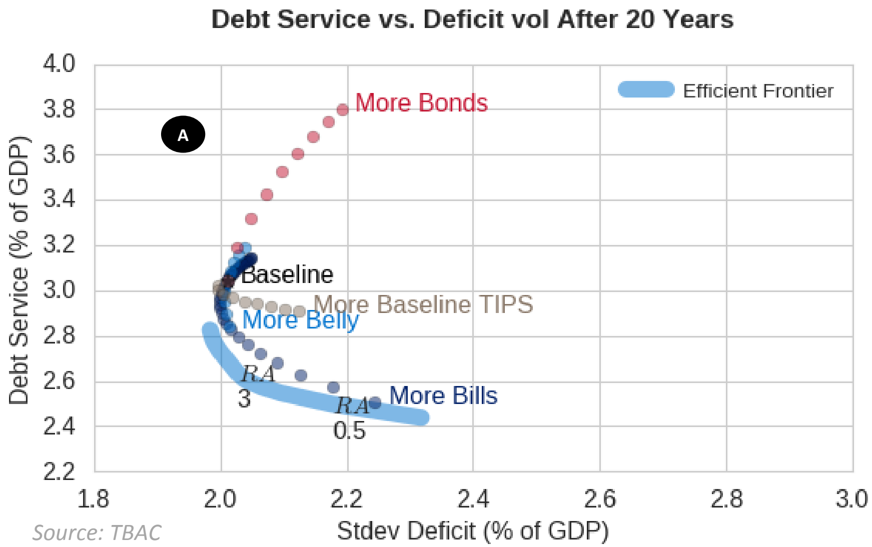
- In the plots above we display the effect of adding progressively more of each kernel to the baseline issuance (defined as one unit of nominal base kernel).

A The results of the "More Bills", "More Belly", and "More Bonds" kernels closely correspond with the results of the previous model.

B Adding more Baseline TIPS decreases cost.

Static Optimization of Kernels

Optimizing over kernel weights produces more realistic issuance strategies



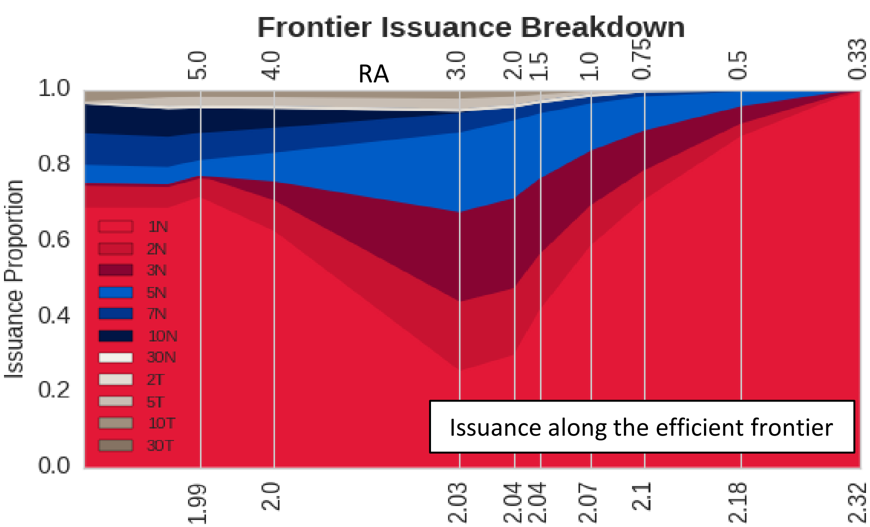
Source: TBAC

A At top left, the efficient frontier comes from minimizing the objective:

$$cost + RA \times risk$$

for different levels of risk aversion (RA).

- The risk aversion coefficient tells the optimizer what the relative importance of cost and risk are to Treasury.
- The two extremes are $RA = \infty$ and $RA = 0$. When RA is large the optimizer focuses almost exclusively on risk reduction and if RA is small the optimizer puts more emphasis on cost reduction.
- The optimizer solves for kernel weights constrained so that issuance proportions are non-negative.
- Gross issuance is zero for all but the base kernel.



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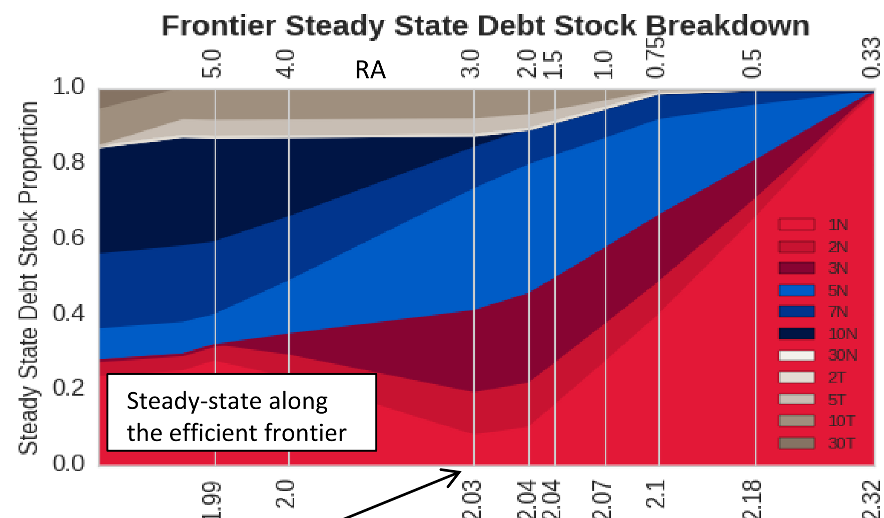


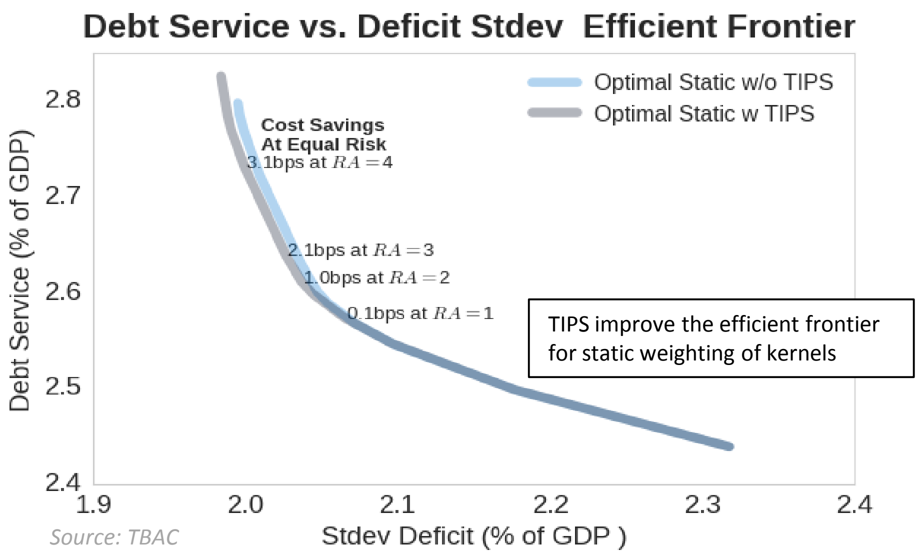
Source: TBAC

TIPS proportion is 12.5% at lower risk end of plausible range.

Plausible Range

Cost Benefit From Issuing TIPS

Optimal allocations include TIPS for a wide range of risk preferences



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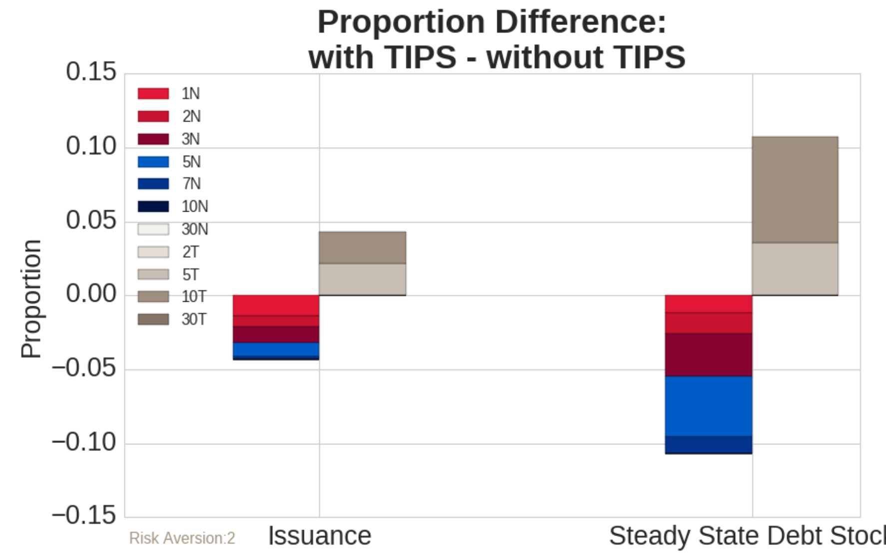

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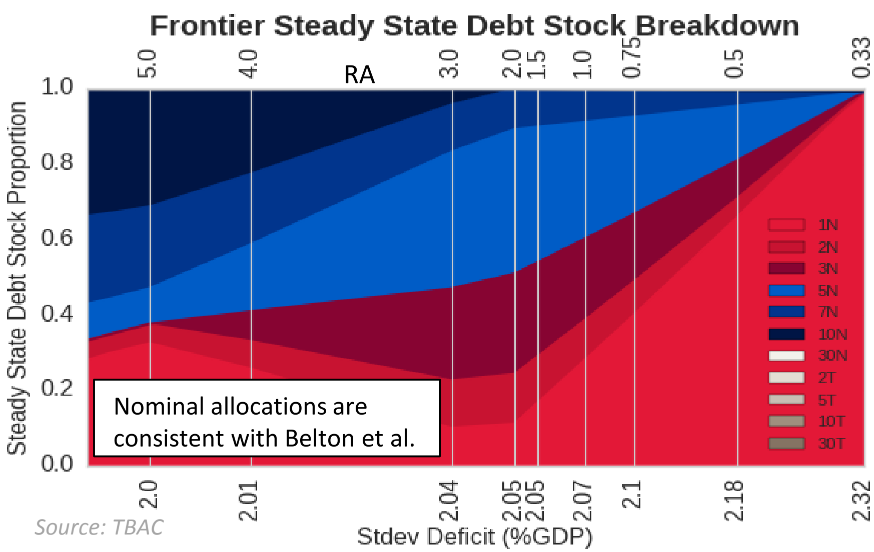

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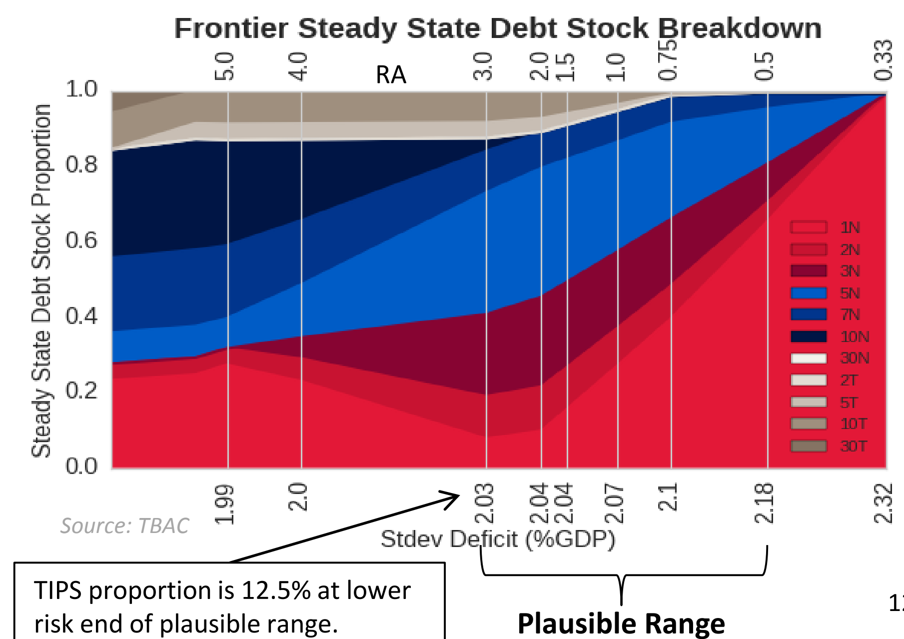

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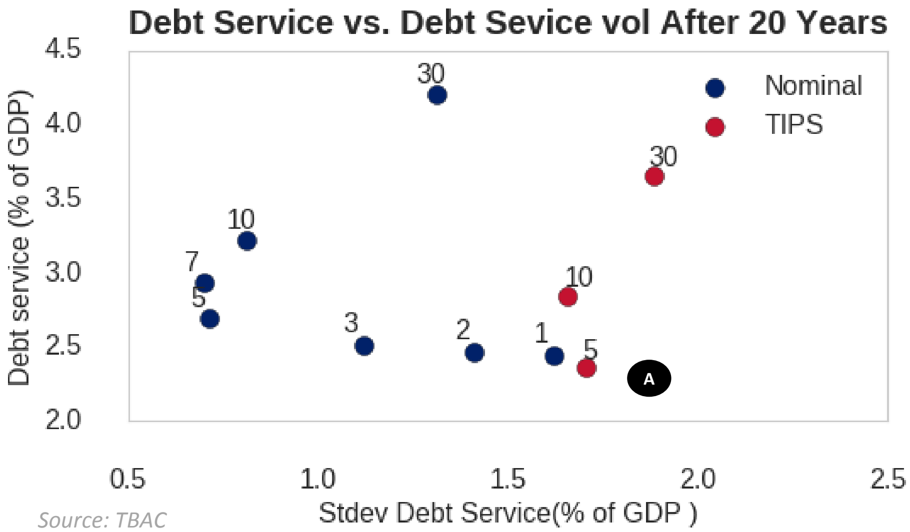
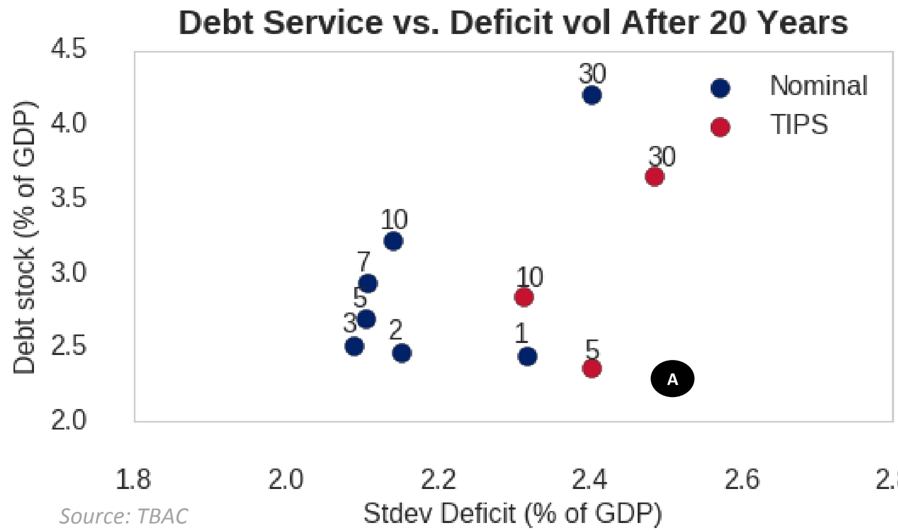
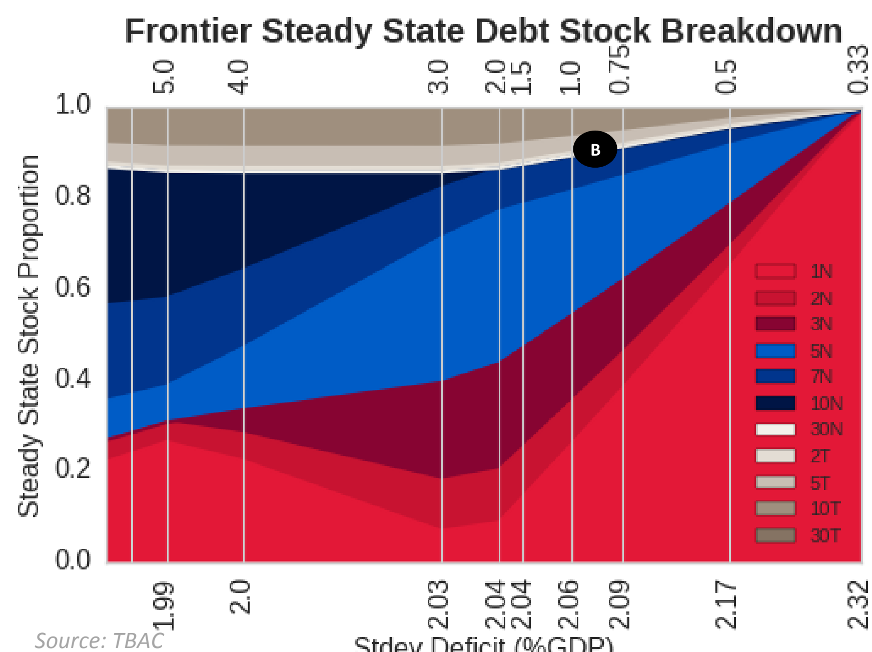
Reassessing 5 Year TIPS Using Alternative LRP

5y TIPS look more attractive if the asset swap market is used to estimate the LRP instead of the ARTS model







TIPS proportion ranges from 2.5% to 14.25% over the plausible range.

Plausible Range

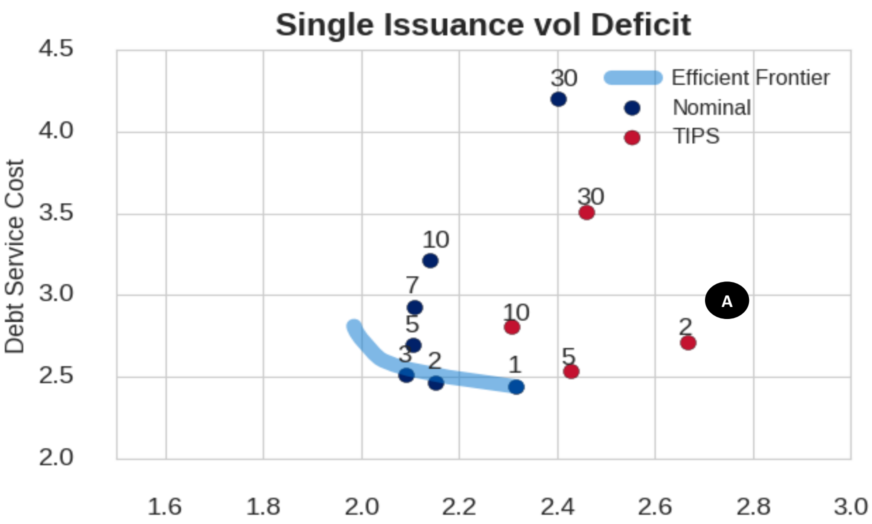
(A) Lowering the liquidity premium for shorter dated TIPS makes them more attractive.

(B) Static kernel-based optimization shows a larger allocation to 5y and 10y TIPS in the range of plausible risk preferences, particularly for higher risk tolerances.

- The relative attractiveness of TIPS versus nominal Treasuries can be similarly shifted by changing assumptions for the long term average level of inflation risk premium.

Assessing 2 Year TIPS in the Model

Treasury doesn't currently issue 2y TIPS; however, the model would provided ASW spreads are used for LRP



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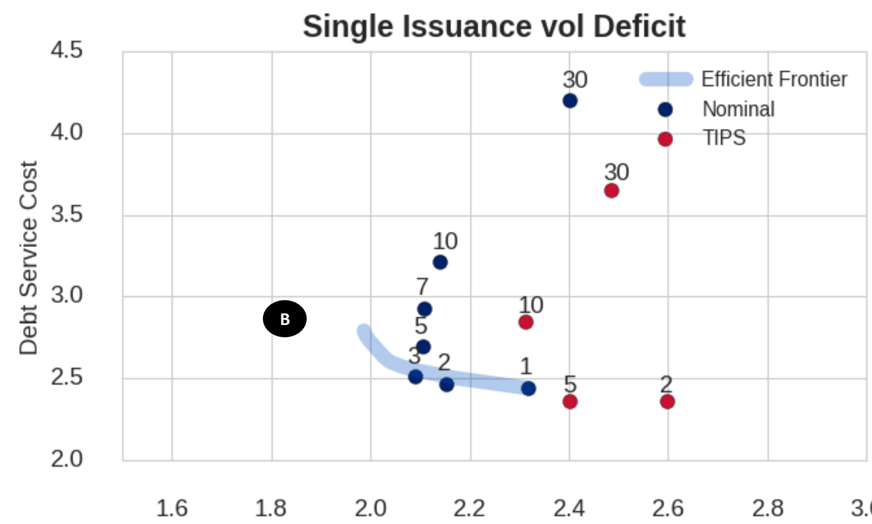

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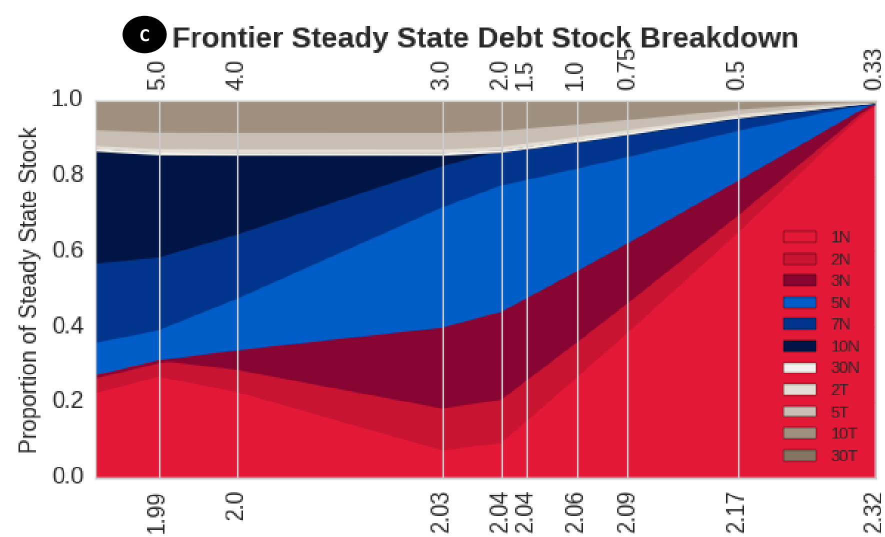
- While Treasury does not currently issue 2y TIPS, we explore their attractiveness in the model.

A Currently, 2y TIPS have a higher cost than 5y TIPS, and they are also significantly more volatile.

B Taking our LRPs from asset swaps (ASW) drops and flattens the 2y-5y LRP (page 6, dotted red line), lowering the cost for those two assets.

C We add 2y TIPS to our More Front TIPS kernel as 20% of issuance and run our model using ASW LRPs.

- Front TIPS issuance becomes optimal at lower levels of risk aversion when using ASW spreads for LRP, due to lower cost of the front-end TIPS.
- A small change in IRP term structure would be equivalent to a relative change in LRP.



Source: TBAC

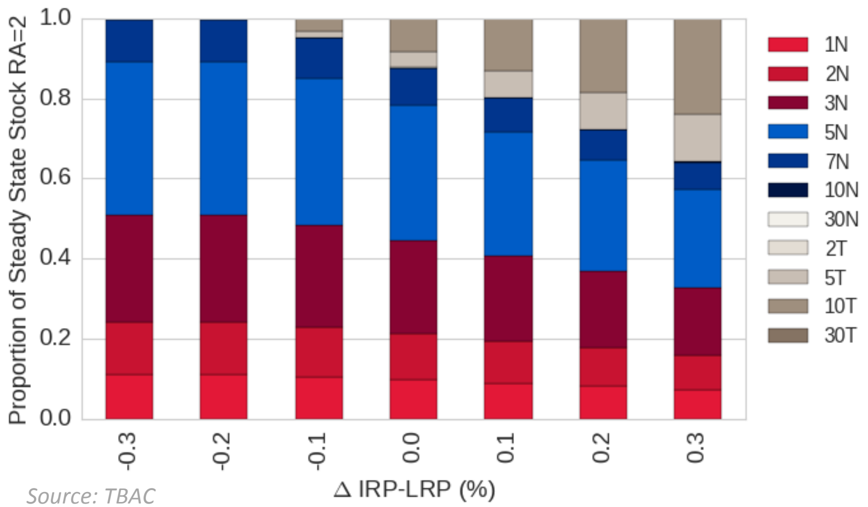
TIPS proportion ranges from 4.5% to 14.33% over the plausible range.

Plausible Range

Examining Effects of IRP, CPI, Correlation

We vary the key drivers of relative cost and risk between TIPS and nominals

A IRP-LRP spread is a key driver of relative cost[1]



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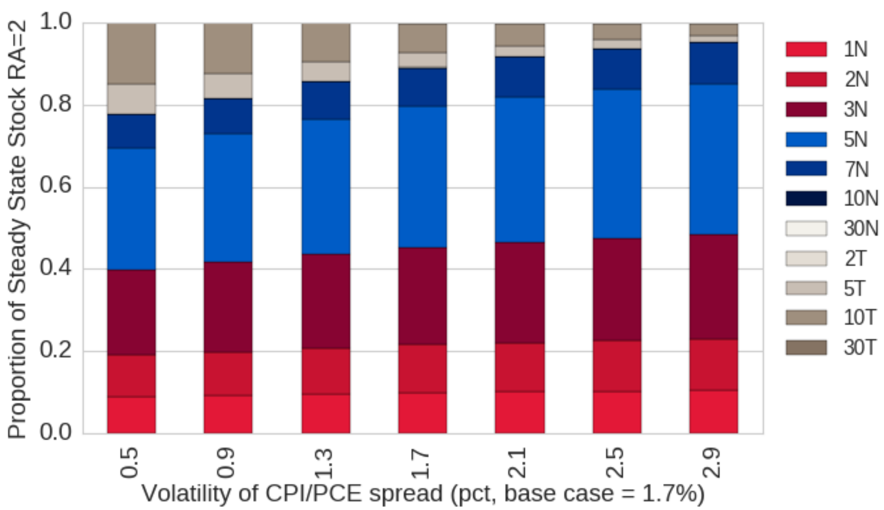
[1] 0.3% change in IRP is roughly the 25/75 percentile range in our simulations (page 5)

- In the cost vs. risk tradeoff of our objective, each component has one key variable which drives the relative attractiveness of TIPS compared to nominals.
- **A** The TIPS cost advantage for the issuer comes from the IRP-LRP spread. At top left, we show the effect of shifting the IRP-LRP spread in parallel across all tenors.
- On the risk side, the TIPS disadvantage is driven by the volatility of CPI, which we model as a spread to PCE.
- **B** The spread volatility is 1.7% and PCE vol is 0.79%, and the two are uncorrelated. At bottom left, we vary the spread vol.
- **C** TIPS inflation indexation helps to lower total deficit vol, because inflation and primary deficit are negatively correlated.

B CPI vol is the main driver of extra TIPS volatility



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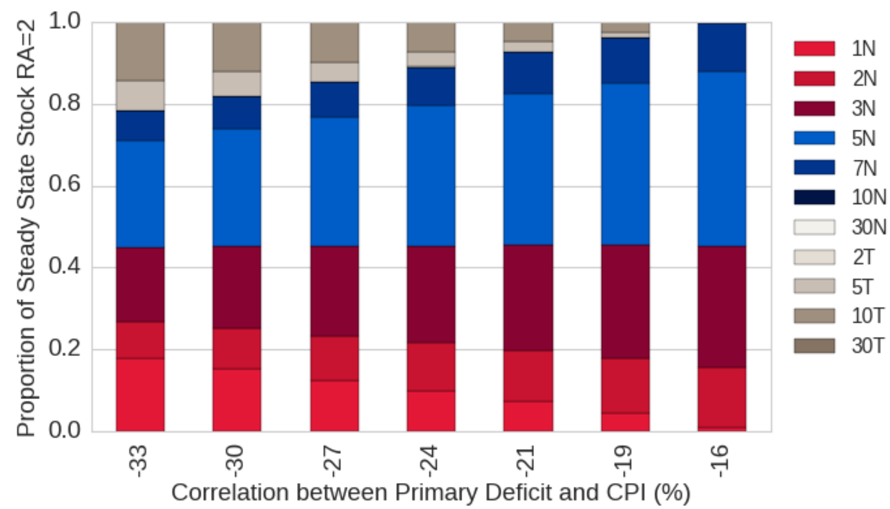
C Less negative correlation makes TIPS less attractive



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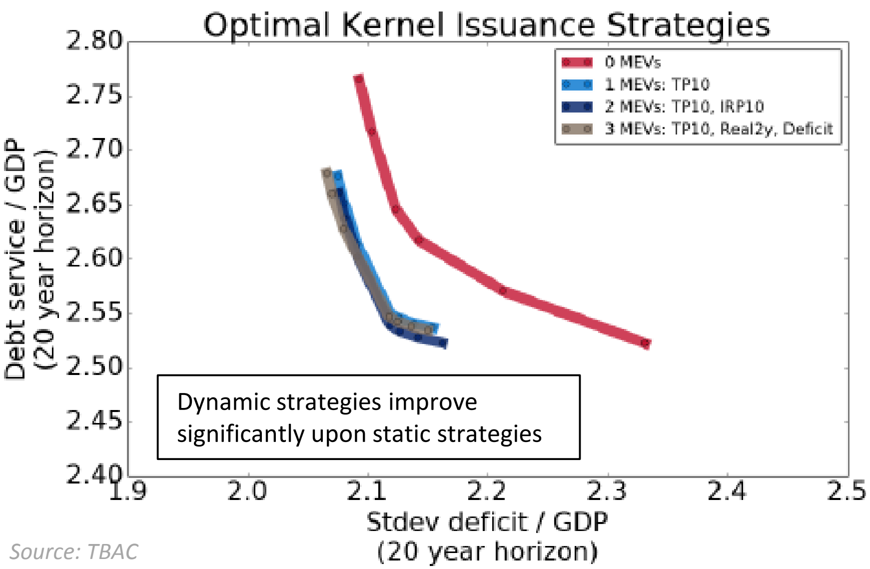
Dynamic Strategy Results

Bootstrap results suggest a sparse set of macro economic variables (MEV) consisting of just IRP10 and TP10



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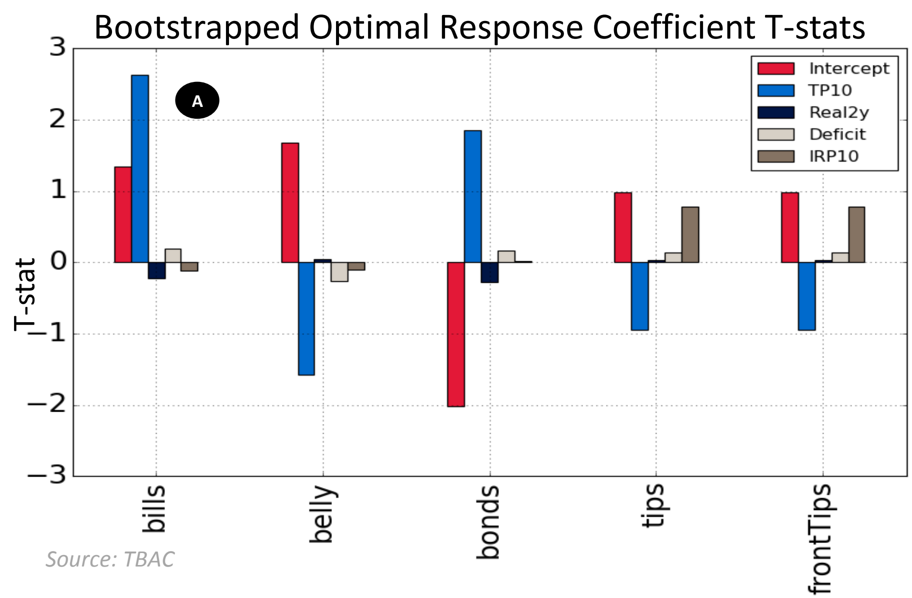


Source: TBAC

- We computed bootstrap t-stats for the optimal response function coefficients:
  - We generated 100 independent simulations, each consisting of 50 paths.
  - For each simulation we estimated the matrix of optimal response coefficients for a risk aversion parameter of 1.
  - For each coefficient we computed the average and standard deviation across the 100 simulations.
  - Finally we set the t-stat of each coefficient to be the ratio between its average and its standard deviation.

A T-stat results suggest that Deficit and Real2y might not be significant; however, TP10 and IRP10 appear significant.

  - Similar to Belton et al., we find that the model rotates out of the belly and into bills as TP10 increases.
  - Additionally, as IRP increases, the model rotates out of bills and into the belly and TIPS.

Bootstrapped Optimal Response Coefficients

	intercept	TP10	Real2y	Deficit	IRP10
bills	7.7%	10.1%	-2.3%	1.1%	-3.4%
belly	8.0%	-1.8%	0.4%	-0.2%	0.6%
bonds	-2.1%	0.5%	-0.1%	0.0%	-0.1%
tips	4.3%	-0.6%	0.0%	-0.3%	0.9%
frontTips	4.3%	-0.6%	0.0%	-0.3%	0.9%

Bootstrapped Optimal Response T-stats

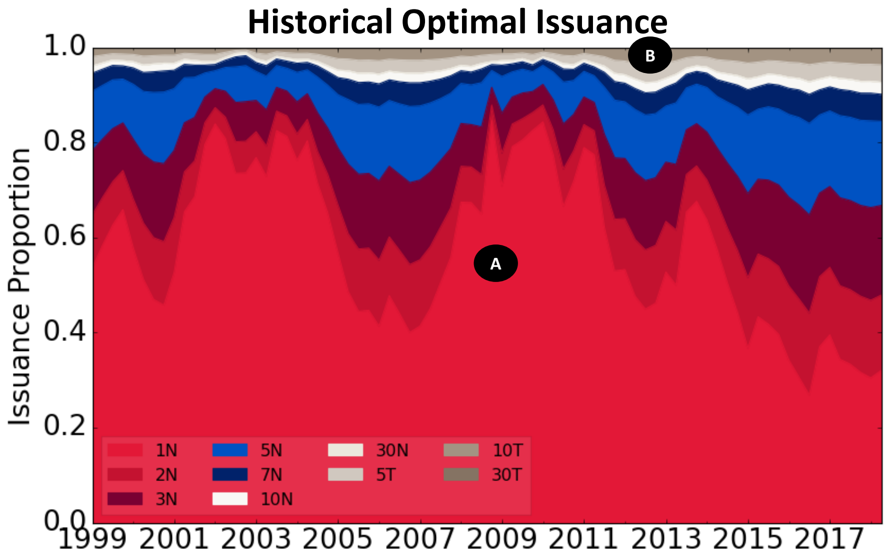
	intercept	TP10	Real2y	Deficit	IRP10
bills	1.38	2.59	-0.23	0.16	-0.09
belly	1.72	-1.55	0.05	-0.23	-0.13
bonds	-2.09	1.80	-0.29	0.13	0.04
tips	0.98	-0.93	0.02	0.13	0.79
frontTips	0.98	-0.93	0.02	0.13	0.79

Source: TBAC

The reaction function is fit to standardized MEVs; therefore, each column of coefficients above represents the effect of a one-sigma move in the corresponding MEV.

Dynamic Strategy Results[1]

Most of the variation in issuance patterns is caused by fluctuations in TP10



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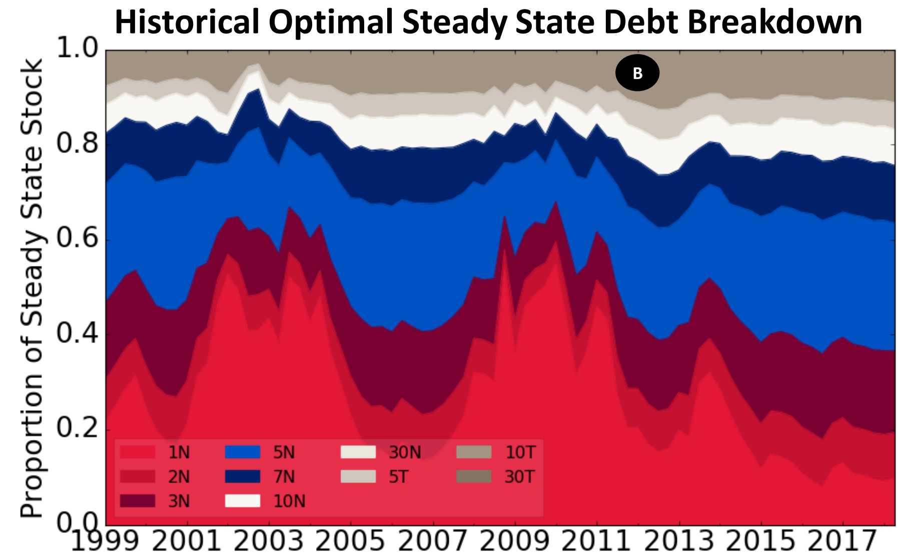
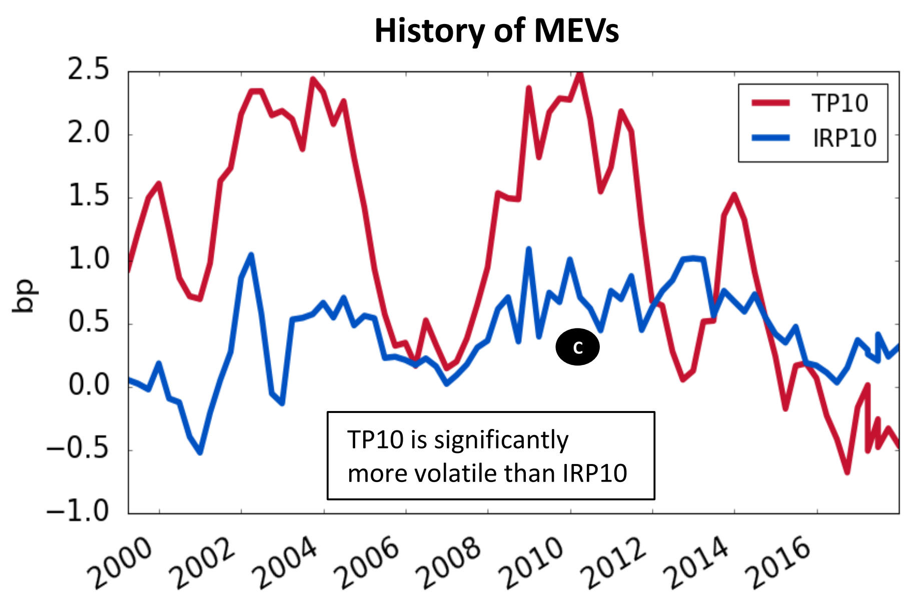


Source: TBAC

- **A** Consistent with the results of Belton et al., most of the fluctuation in issuance comes via bills and belly kernels.

- **B** TIPS issuance ranges from 1% to 7%, with an average of 4%. The steady state proportions range from 5% to 19%, with an average of 13%.

- **C** TP10 is equal to RRP10 + IRP10. Most of the variation in TP10 is coming from fluctuations in RRP10.

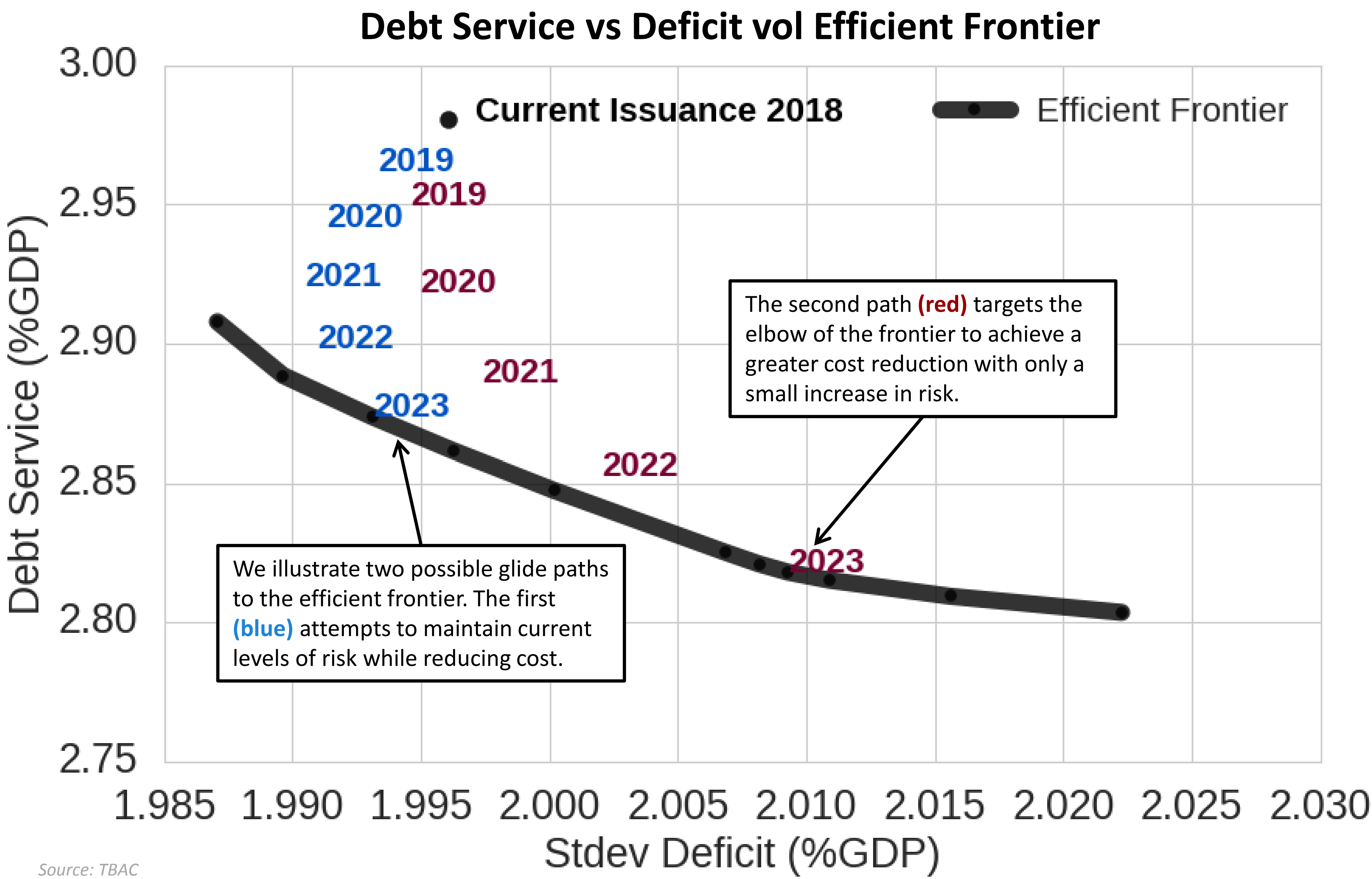


Source: TBAC

[1]Back test uses a risk aversion parameter of 2 and imposes 0% lower bounds on issuance sizes.

Case Study: Optimizing 2019 Issuance

We use the model to build an efficient frontier for issuance while maintaining minimum issuance sizes. We also study glide paths from 2018 issuance weights to the frontier.



Source: TBAC

Case Study: Two Glide Paths Toward Lower Cost Issuance

- A The **blue** glide path of issuance presented on the previous slide decreases cost while maintaining or reducing the level of risk associated with current issuance patterns. This path toward the frontier steadily increases allocations to bills and TIPS at the expense of the all other issues.

- B The **red** glide path of issuance aims toward the elbow of the efficient frontier (a point with a good cost to risk tradeoff). With each step along the path, allocations to the belly and TIPS expand, while allocations to bills and the long end shrink.

- A blend of these two allocations could be used to move closer to the efficient frontier with relatively small absolute changes in issuance sizes.

Issuance glide path: blue

Issuance Proportion Through Time (%)						
	2018	2019	2020	2021	2022	2023
Debt Service	2.980	2.962	2.942	2.921	2.898	2.874
Stdev Deficit	1.996	1.993	1.991	1.990	1.991	1.993
Stdev Debt Service	0.734	0.751	0.771	0.795	0.824	0.858
Bills + FRN A	53.7	56.5	59.2	62.0	64.8	67.6
2y Nominal	8.7	8.2	7.6	7.1	6.6	6.1
3y Nominal	8.2	7.5	6.8	6.1	5.4	4.7
5y Nominal	9.2	8.6	7.9	7.3	6.6	6.0
7y Nominal	7.7	7.1	6.4	5.8	5.1	4.5
10y Nominal	5.9	5.6	5.3	4.9	4.6	4.3
30y Nominal	3.9	3.5	3.1	2.8	2.4	2.0
TIPS	2.8	3.2	3.6	4.0	4.5	4.9

Source: TBAC

Issuance glide path: red

Issuance Proportion Through Time (%)						
	2018	2019	2020	2021	2022	2023
Debt Service	2.980	2.950	2.919	2.886	2.853	2.818
Stdev Deficit	1.996	1.995	1.995	1.997	2.002	2.009
Stdev Debt Service	0.734	0.758	0.787	0.821	0.860	0.904
Bills + FRN B	53.7	53.1	52.6	52.0	51.4	50.9
2y Nominal	8.7	9.1	9.4	9.8	10.2	10.6
3y Nominal	8.2	9.4	10.7	11.9	13.2	14.4
5y Nominal	9.2	9.9	10.5	11.1	11.7	12.4
7y Nominal	7.7	6.7	5.7	4.7	3.7	2.8
10y Nominal	5.9	4.9	3.9	3.0	2.0	1.0
30y Nominal	3.9	3.6	3.3	3.0	2.7	2.4
TIPS	2.8	3.4	3.9	4.5	5.1	5.6

Source: TBAC

Limitations

Modeling Considerations

- This work represents one model with results that depend critically on model assumptions. TBAC does not drive recommendations off of one model, but instead takes into account a wide range of inputs on investor demand and market pricing.
- Results depend critically on the choice of risk measure (standard deviation of deficit versus debt service), and correlation between primary deficits and inflation.
- Results depend heavily on the ex-ante assessment of term premium and its decomposition into inflation, liquidity, and real risk premia.
- Results depend heavily on debt manager risk aversion.

Investor Demand Considerations

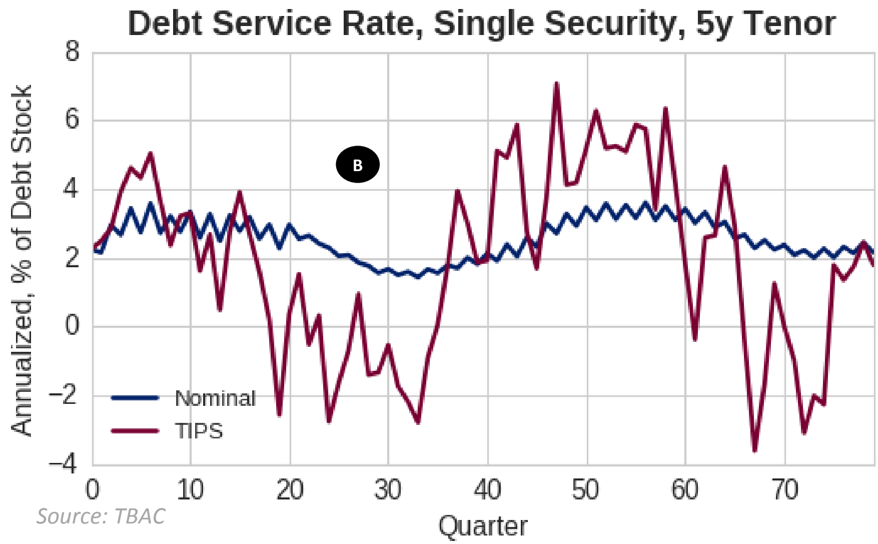
- TIPS trading volumes and turnover suggest that they are less liquid than nominal Treasuries and Conventional MBS. This may be due to the lack of an active derivatives / futures market.
- TIPS are more complex than nominal Treasuries.
- For tax purposes, TIPS are treated as original issue discount (OID) bonds, which means that increases in TIPS principal are taxable for the year in which they occur, rather than at maturity.

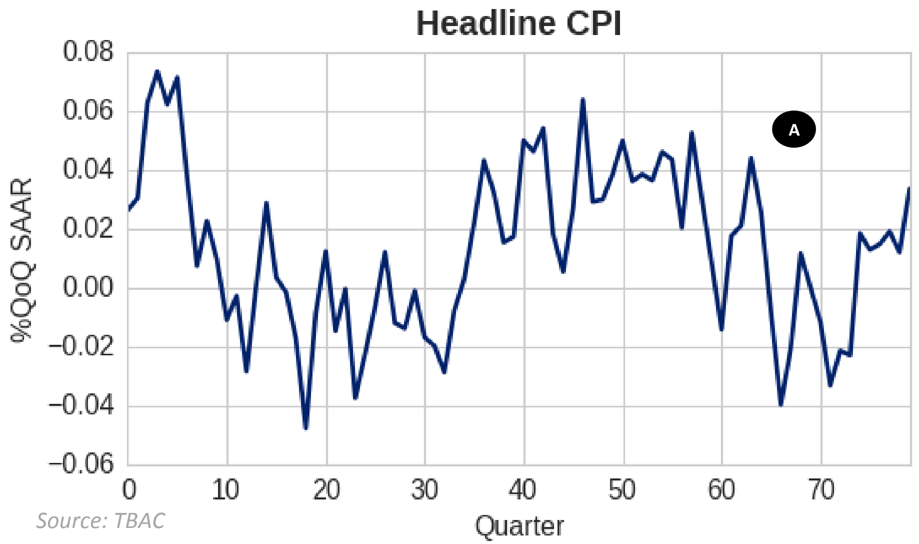
Conclusions and Recommendations

- The extension of the model of Belton et al. to incorporate TIPS demonstrates a cost and risk reduction for the issuance of TIPS in addition to nominals.
    - Assuming historical correlations hold, total deficit volatility is reduced for TIPS allocations up to 13% of the debt stock (currently TIPS make up 9% of the debt stock).
- The optimal amount of TIPS to issue varies based on choice of risk metric, assessment of market risk premiums, and Treasury's overall risk appetite.
- Given the diversification / correlation benefits, as well as the benefits of having benchmark issuance across the entire curve, continued issuance across the existing benchmark tenors (5y, 10y, 30y) is appropriate.
    - The analysis of potential issuance of 2y TIPS illustrates that benefits here may be more limited, but further study is needed.
- The model finds that TIPS dependence on CPI causes them to behave like floating rate notes, and thus have many of the same risk characteristics as bills (both are relatively lower cost and higher volatility), but further study is needed.
- Overall, though further work is still needed, the model does correspond well with market intuition and provides a useful framework for future analysis of the tradeoffs involved in achieving a more optimal issuance allocation.

Appendix

TIPS Interest Expense Accounting





INTEREST EXPENSE ON PUBLIC ISSUES (C)			
ACCRUED INTEREST EXPENSE			
	Treasury Notes		$11,131,125,626.39
	Treasury Bonds		$5,881,996,867.71
	Inflation Protected Securities (TIPS)		$892,576,159.49
		Int. Expense Inflation Compensation (TIPS) (D)	($5,635,577,715.66)
	Treasury Floating Rate Notes (FRN)		$11,030,196.93
	Domestic Series - C/I's & Demand Deposits		$0.00
	Foreign Series - C/I, Notes & Bonds		$0.00
	REA Series		$0.00
	State & Local Government-C/I's, Notes & Bonds		$87,837,116.27
	Matured Debt		$0.00
		TOTAL ACCRUED INTEREST EXPENSE	$12,368,988,251.13

A Treasury records the principal accrual of TIPS as an interest expense (or interest income) according to moves up (down) in CPURNSA

B We follow this treatment in our simulation block, resulting in the behavior shown above

C Treasury reporting for month of February 2015

- Uses CPURNSA change from mid-Nov to mid-Dec 2014
  - 2014 Refs: mid Nov 236.792, mid Dec 235.4815
  - Change – 0.5534%

D Treasury interest credit of $5.636Bn

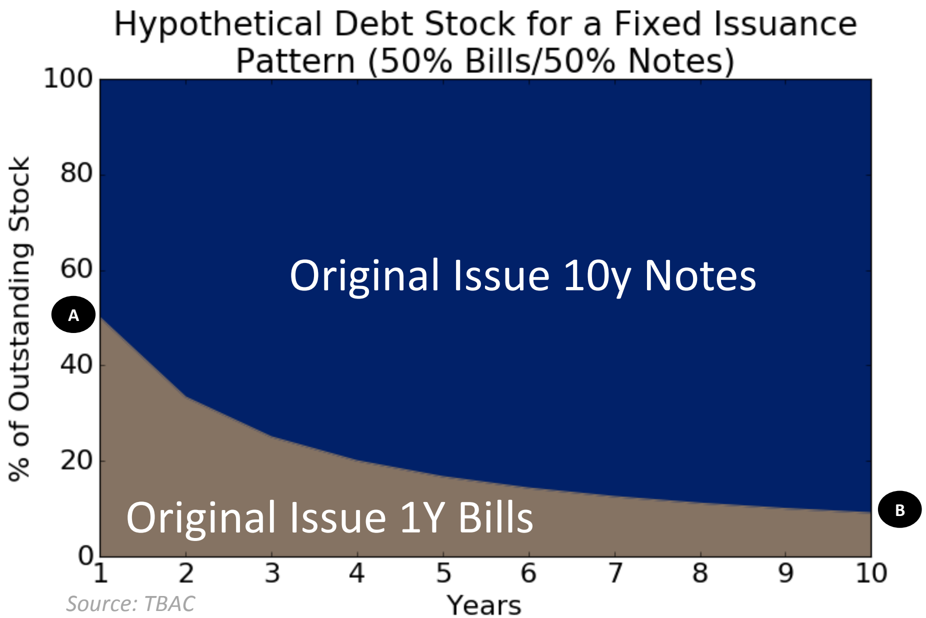
- Implies outstanding TIPS notional of $5.636Bn/0.5534%
  - Implies $1.02Tn TIPS outstanding in Feb2015
  - Bloomberg DEBPINNT Index: $1.07Tn TIPS outstanding

See, for example, https://www.treasurydirect.gov/govt/reports/ir/IntExp022015.xls

Maturity Weighted Issuance

We focus on maturity weighted issuance because it more closely aligns with steady state portfolio metrics

- A Consider a hypothetical issuance split 50% : 50% between 1y Bills and 10y Notes.
- In steady state, 100% of the outstanding stock of Bills turns over every year, but only 10% of the stock of 10y Notes would be redeemed.
- B The 50% : 50% issuance split leads to a 9% : 91% Bills / Notes steady state distribution.
- The weighted average maturity of the steady state debt distribution is 4.6 years, which is more than ½ the WAM of a 1Y + the WAM of a 10y (2.75 years).



Source: TBAC

- Suppose Treasury can issue securities with maturities $\{\tau_1, \ldots, \tau_M\}$. Denote by w_m the fraction of each years debt issued in the m-th maturity with $\sum_{m=1}^{M} w_m = 1$.
- Assume that quarterly issuance is a constant one unit, and that the issuance fractions never change. Then after a long time, the total amount of outstanding debt which is an original-issue τ_m- maturity security is simply $\tau_m w_m$, because it takes τ_m years for each w_m of debt issued to mature. The total stock of debt is simply $D = \sum_{m=1}^{M} \tau_m w_m$.
- We can define the *steady-state debt stock fractions*

$$\bar{w}_m = \frac{\tau_m w_m}{\sum_{n=1}^{M} \tau_n w_n},$$

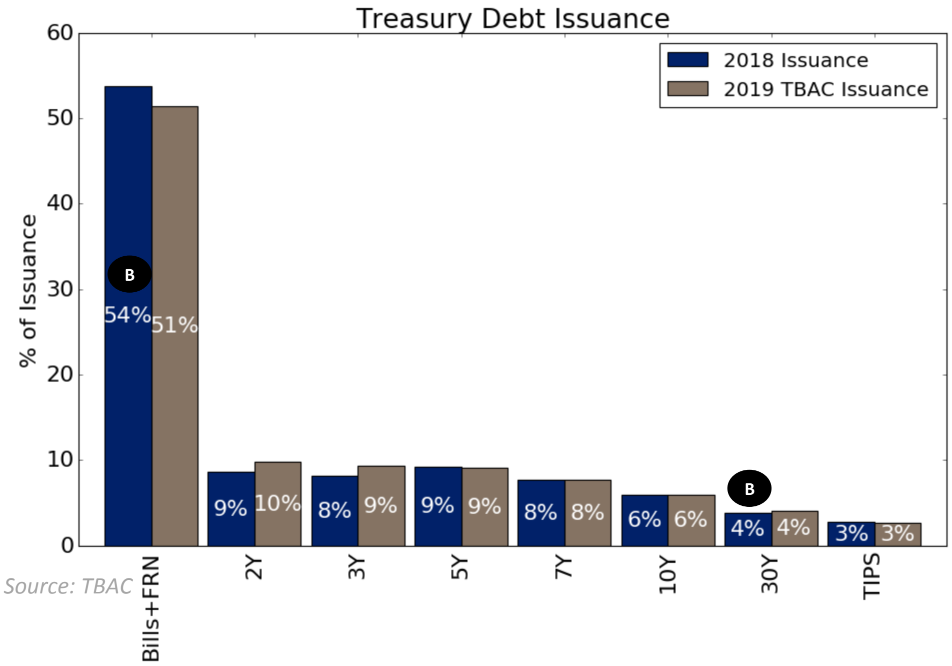
which also sum to 1. The weighted average maturity of the debt stock can be computed as $W = \frac{1}{2} \sum_{m=1}^{M} \bar{w}_m \tau_m$.

- The relationship can also be inverted, so that if one has a desired set of steady state debt stock fractions, one can find the required yearly issuance fractions as

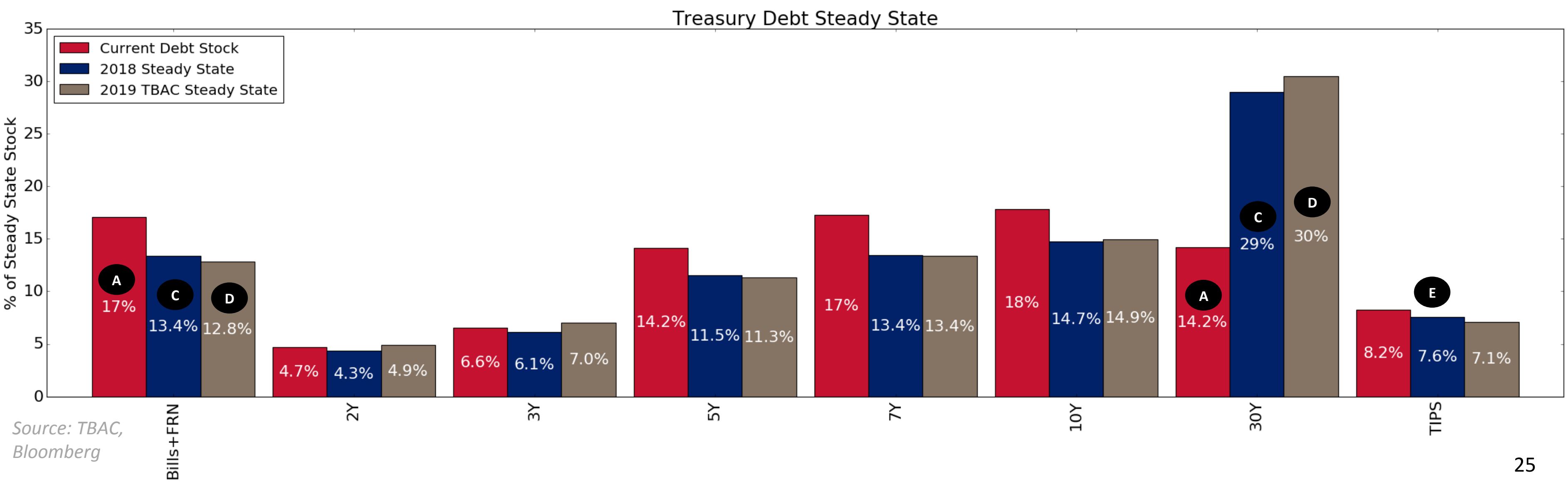
$$w_m = \frac{(\bar{w}_m / \tau_m)}{\sum_{n=1}^{M} (\bar{w} / \tau_n)}.$$

Steady State Based on Current/Projected Issuance

- **A** As a point of reference, the current debt stock has approximately the same allocation of original issue bills and 30s (17% and 14% respectively).
- **B** In terms of 2018 issuance, bills far outweigh bonds (54% and 4% respectively).
- **C** If 2018 issuance percentages are held constant, the steady state allocations will converge to 29% for bonds and 13.4% for bills.
- **D** With increasing percentages of 30y issuance projected for 2019, the steady state stock of 30y will be even higher (30%) and the bills slightly lower (12.8%).
- **E** Based on 2018 maturity issuance in TIPS, the stock will fall through time, from the current level of 8.2% to 7.6%.



Source: TBAC



Source: TBAC, Bloomberg

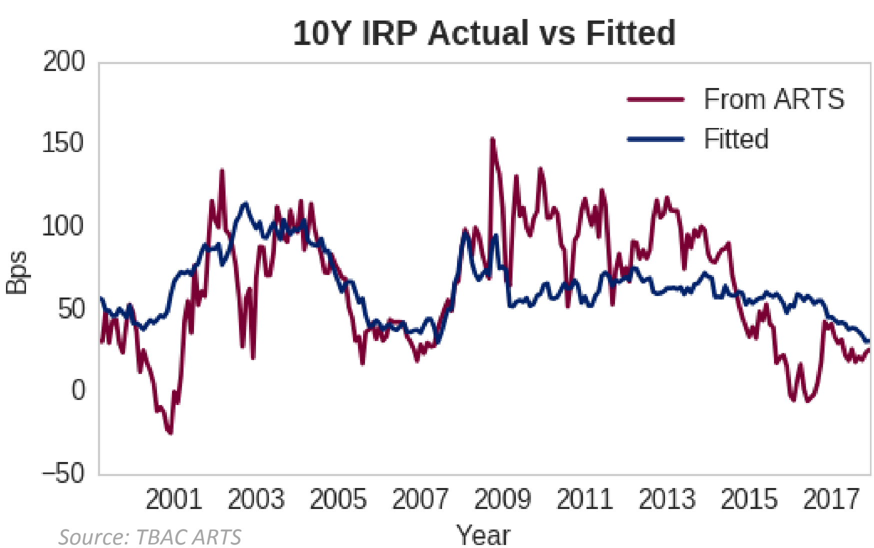
Inflation Risk Premium Regression Details

We fit historical inflation risk premia from our implementation of the AACM model (ARTS) to the historical expected level of monetary policy accommodation:

$$IRP_{t,\tau} = \alpha_\tau + \beta_\tau\left(r^P_{t,\tau} - r^*_t\right) + \epsilon_{t,\tau}$$

where

- $IRP_{t,\tau}$ is the τ year inflation risk premium from our model
- $r^P_{t,\tau}$ (expected real rate) is the τ year nominal yield less the τ year ACM TP less expected inflation[1]
- r^*_t is the neutral real rate of interest
- t is time through history



Source: TBAC ARTS

- In our simulation block we model 5y and 10y IRP as an affine function of $r^P_{t,\tau}$ - r^*_t, where both rates are in the block.
- We take the betas from the historical regression.
- We choose intercepts to match steady-state levels to their five-year averages in the ARTS model.
- Residual AR1 processes come from the historical regression

$$IRP_{t,5} = 0.61 \; - 0.145\left(r^P_{t,5} - r^*_t\right) + \epsilon_{t,5}$$

$$IRP_{t,10} = 0.61 \; - 0.245\left(r^P_{t,10} - r^*_t\right) + \epsilon_{t,10}$$

where t is now the forward time of our simulation

[1]We calculated the historical time series of expected inflation using the method described in Belton et al.

Simulation Module Outputs