Origins of the Yield Curves

James A. Girola

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Introduction

- This presentation describes the origins of the suite of yield curves produced in the Office of Economic Policy.
- The first yield curve was the High Quality Market (HQM) Corporate Bond Yield Curve, which is mandated by the Pension Protection Act of 2006.
- Subsequently, Economic Policy produced the Treasury Nominal Coupon-Issue (TNC) Yield Curve, which was followed by the Treasury Real Coupon-Issue (TRC) Yield Curve for TIPS and the Treasury Breakeven Inflation (TBI) Curve providing breakeven inflation rates.
- The presentation sets out the purposes of the curves and the challenges faced in their construction that required new methodology.

The Pension Protection Act

- The Pension Protection Act of 2006 (PPA), signed into law on August 17, 2006, implements reforms that improve pension plan funding.

- The PPA contained the significant innovation that discount rates for computing the funding of single-employer pension plans would be given by a yield curve rather than by a single interest rate. Specifically, actuarial estimates of future pension outlays would be discounted by spot interest rates of the same maturities that would be derived from a yield curve for high quality corporate bonds.

- The purpose of the yield curve is to take account of differences among pension plan participants: a younger workforce would have longer-term spot rates because outlays would be later.

Need for New Methodology

- However, at the time when the PPA was under consideration, it was clear that existing yield curve methodologies were not up to the task of producing a usable yield curve:
  - Available yield curve approaches had limited foundation in bond market characteristics, giving different results with no way to choose among them. Therefore, results were arbitrary;
  - Available approaches frequently produced unacceptable negative nominal yields, often at long maturities, as well as unstable yields with spurious movements, all of which were inconsistent with market observations;
  - And a common problem among the approaches was numerical ill-conditioning which either gave multiple answers or didn’t converge at all.

Additional PPA Challenges

- Proposed PPA requirements added additional challenges to the yield curve:
  - The PPA required a corporate bond yield curve while yield curves at the time focused on Treasuries. Corporates have many complexities that would have caused more large problems with available approaches;
  - Moreover, the PPA required that AAA, AA, and A bonds be combined into a single weighted-average yield curve: there was no approach to do this;
  - The PPA also needed spot rates for discounting beyond the maximum bond maturity of 30 years. Again, there was no approach to do this, and at that time spot rates above 30 years maturity were usually held constant at the 30-year rate which resulted in inaccurately high discounted values.

Purpose of the HQM Yield Curve

- The purpose of the HQM yield curve was to solve all these problems, both problems of generic yield curve approaches and problems stemming from PPA requirements, and to provide a new yield curve methodology and new spot discount rates that would be accepted by the business community and the pension industry and would enable the PPA to become law.

- Some observers at the time thought that creating such a yield curve that would achieve consensus was impossible and that the PPA yield curve proposal would fail. Nevertheless, the HQM methodology has succeeded and is accepted by the private sector and in government.

- The following discussion outlines the HQM methodology.

The Discount Function

- There are different ways to approach the estimation of a yield curve. The HQM methodology follows the approach often seen of estimating a bond price equation and calculating spot rates and other results from the equation estimates.

- The starting point for the bond price equation is the discount function $\delta(\tau)$, which gives the price today of $1 to be received τ years in the future.

- Using the discount function, the price of a bond in the HQM methodology is given by the following equation. However, the HQM price equation also includes regression variables in addition to the discount function; other yield curve approaches don’t have regression variables:

The Bond Price Equation

$$p + a = \sum_{i=1}^{n} \delta(t_i) c_i + \sum_{j=1}^{m} \vartheta_j x_j + \varepsilon$$

where:

- p is the (flat) price of the bond and a is accrued interest;
- c_i, $i = 1, \ldots, n$, are the n cash flows from the bond to be received at maturities t_i;
- ϑ_j, $j = 1, \ldots, m$, are the coefficients on the m regression variables x_j;
- ε is the error term.

Yield Curve Estimation

- With the price equation, the process of yield curve estimation focuses on the discount function. The difference between HQM and other approaches is the method of determining the discount function as well as the inclusion of possible regression variables.

- The next slides describe the HQM methodology for the discount function. The methodology is based on the concept of maturity ranges that are grounded in established bond market characteristics and that ensure that the resulting mathematical form of the yield curve is not arbitrary.

- It's important to note that the HQM methodology achieves its results by making appropriate assumptions about bond market behavior. Perhaps the main reason why other approaches often encounter difficulties is that they usually try to avoid making assumptions.

The Forward Rate

- However, rather than estimating the discount function directly, it's more convenient to work with the forward rate $\varphi(\tau)$. For a loan to be repaid at a given maturity, the forward rate at that maturity is the future interest rate for extending the loan for a short period.

- The forward rate is higher at a given maturity when investors who are trading at that maturity are less eager to lend based on their assessment of uncertainty and their expectations and purposes while borrowers are more eager to borrow based on their perceptions. The forward rate summarizes market views for each maturity in a single number.

- The discount function and forward rate can be determined from each other:

$$\varphi(\tau) = \frac{-d\delta(\tau)}{d\tau}\frac{1}{\delta(\tau)}$$

Maturity Ranges

- Moreover, trading in bonds divides into maturity ranges, such that the trading activity in each range on average reflects similar purposes, similar views of risk, and similar expectations about bonds in that range.

- Therefore, the forward rates in each maturity range are related to each other. Specifically, the HQM methodology models the forward rates in each maturity range as a cubic polynomial, and the polynomials in the ranges are joined together smoothly resulting in a cubic spline.

- The cubic spline is represented as a set of B-splines that span the space of possible cubic splines with knot points delineated by the maturity ranges. The estimation process provides values for the B-spline coefficients that then give the spot rate, forward rate, and other results.

Choice of Maturity Ranges

- The maturity ranges used for both high quality corporate bonds and Treasury securities are as follows. These ranges accord with market views about trading activity and have worked very well for many decades:
  - First range: 0 to 1-1/2 years maturity, containing the shortest securities;
  - Second : 1-1/2 years to 3 years, including the important 2-year note;
  - Third: 3 to 7 years, including the 5-year note;
  - Fourth: 7 to 15 years, including the benchmark 10-year note;
  - Fifth: 15 to 30 years, including long-term bonds of 20 and 30 years maturity.

Short-Term Constraints

- The HQM methodology includes constraints to reflect market characteristics. The constraints are implemented as linear constraints on the B-spline coefficients: the yield curve software has the ability to impose any linear constraints.

- To ensure that the forward rate moves smoothly to zero maturity, the forward rate is constrained to become linear at zero.

- Moreover, when market short-term interest rates are virtually zero as has been true recently, estimated yield curve short-term forward rates can turn negative. To prevent this, the B-spline coefficients are constrained to be positive.

Consistency Requirement of the Forward Rate

- The forward rate is also constrained to be consistent at 30 years maturity with forward rates before 30 years maturity. Other yield curve approaches don’t have this consistency requirement.

- This requirement eliminates the frequent problem of the forward rate becoming negative near 30 years maturity. In addition, it makes possible projection of the forward rate beyond 30 years maturity out through 100 years maturity, and the projection is in accord with market yields before 30 years maturity.

The Long-Term Forward Rate

- To implement this consistency requirement, the long-term forward rate at and beyond 30 years maturity is fixed at a constant and the forward rate curve is made to flatten out to this constant at 30 years maturity.

- The value of this constant is determined by the same market factors that affect the forward rate in the farthest 15- to 30-year maturity range, because that range is sufficiently distant in time to reveal underlying long-term attitudes toward risk and return.

- Therefore, the constant long-term forward rate from 30 years maturity up through 100 years maturity is set at the average forward rate in the farthest maturity range.

Projection

- Using the long-term forward rate, the yield curve can be projected beyond 30 years maturity out through 100 years maturity, and the projected spot rates from the yield curve provide discount rates for long-dated cash flows.

- Moreover, the method of construction of the long-term forward rate ensures that the projected spot rates are consistent with spot rates of fewer than 30 years maturity and with long-term investment returns available in the market.

Regression Variables

- As indicated in Slide 8, the HQM methodology enables the inclusion of regression variables along with the discount function.
- The addition of regression variables opens many new possibilities for fixed income research. In previous yield curves, the bonds had to be homogeneous, but with regression, bonds with different characteristics can be combined in a single yield curve and the regression terms can account for and estimate the effects of the various characteristics.

HQM Regression Variables

- To fulfill the PPA mandate that the HQM yield curve be a market-weighted average of AAA, AA, and A quality bonds, the HQM methodology includes all three qualities in the curve at the same time and employs two regression variables to account for quality differences.
- Specifically, the discount function is assumed to pertain to market-weighted quality, and the regression variables adjust the price of each bond to give the price the bond would have if it were market-weighted quality. The variables are defined so that bonds whose quality has lesser market weight are adjusted by a greater amount.
- For example, for the month January, 2022 and a $100 bond, a 10-year AAA rated bond’s price is on average 3.34 above the price of a 10-year bond of HQM quality. The 10-year AA price is 0.84 above and the 10-year A price is 0.31 below.

HQM Regression Variables, continued

- The HQM spot rate is then directly calculated from the discount function giving market-weighted quality.

- The inclusion of all three qualities AAA, AA, and A in a single yield curve estimation with regression is analogous to seemingly unrelated regression: here the common market structure of the three qualities is used to provide more efficient estimates of the discount function when they're all estimated together than if separate yield curves were computed for each quality.

Conclusion: HQM Methodology

- This discussion shows that the HQM yield curve solves the yield curve challenges previously set out:
  - The methodology has its foundation in bond market behavior and is not arbitrary;
  - All yields are positive and accord with market observations with no spurious movements;
  - And the methodology is exceptionally stable and very well-conditioned numerically. The custom nonlinear least squares estimation technique uses an algorithm with a line search, but it is so stable that the line search is typically not needed. The algorithm converges in a few iterations from any starting point to the minimum sum of squared residuals.

Conclusion continued: HQM Methodology

- And for the yield curve challenges from the PPA:
  - The methodology pertains to corporate bonds;
  - It includes AAA, AA, and A bonds in a single yield curve using regression variables to account for separate quality effects;
  - And it provides an approach grounded in market behavior that determines the long-term forward rate and projects the spot rate and other yield curve results out through 100 years maturity.

Conclusion continued: HQM Methodology

- After its introduction in the PPA, the HQM yield curve was carried back through 1984 as mandated by the Moving Ahead for Progress in the 21st Century Act of 2012 (MAP-21).

- In addition, along with the HQM yield curve, the companion Treasury BBB (TBBB) Yield Curve is produced internally pertaining to BBB corporate bonds. The HQM and TBBB yield curves together provide the yield curve for investment grade corporate bonds.

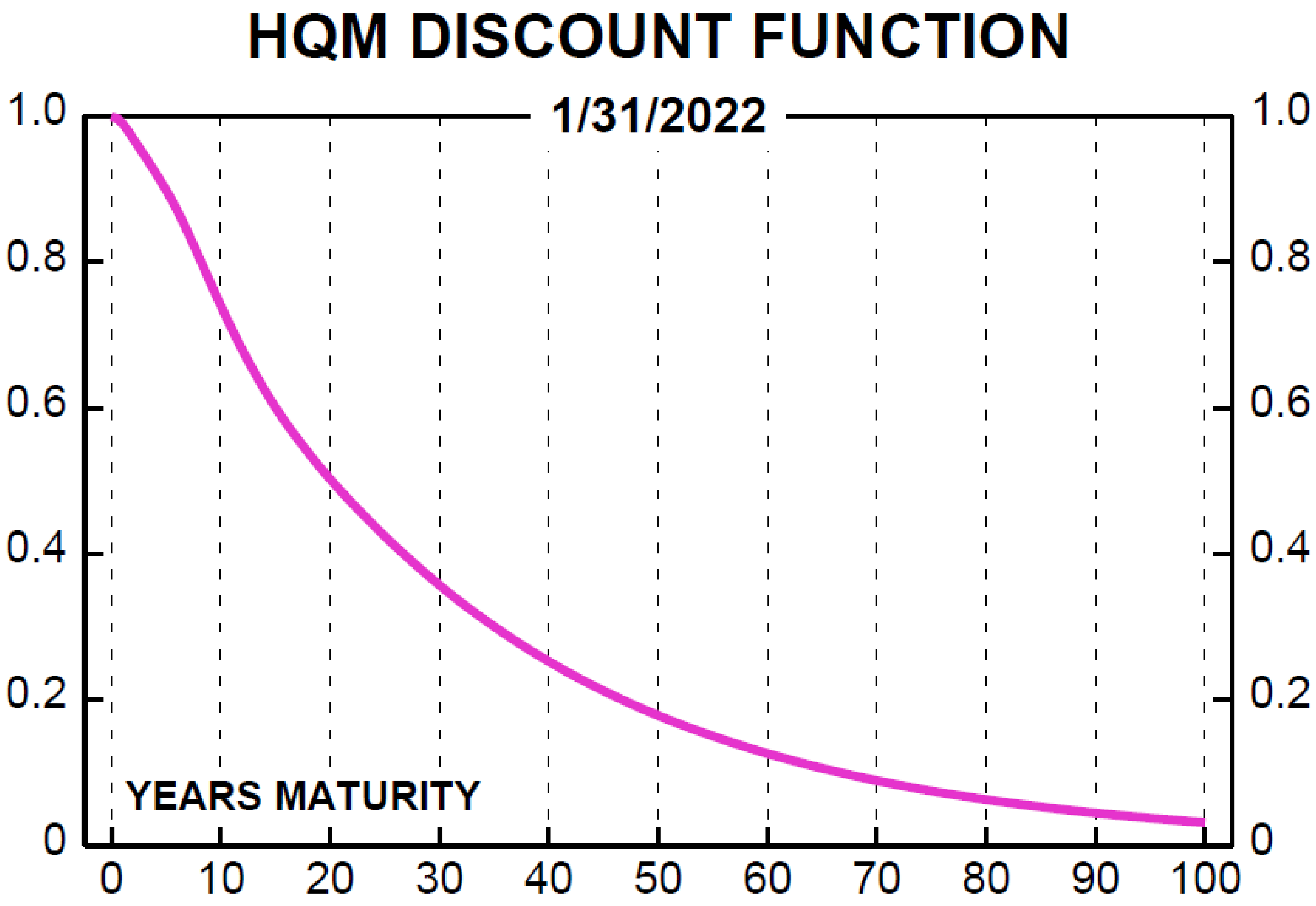
HQM Discount Function and Forward Rate, January 31, 2022

- The next two slides show the discount function and forward rate from the HQM yield curve for the last day of January 2022 projected out through 100 years maturity. The positive values of the spline coefficients for nominal bonds guarantee that the forward rate is positive and the discount function declines continuously. The discount function starts at unity because the price of $1 received now is $1.

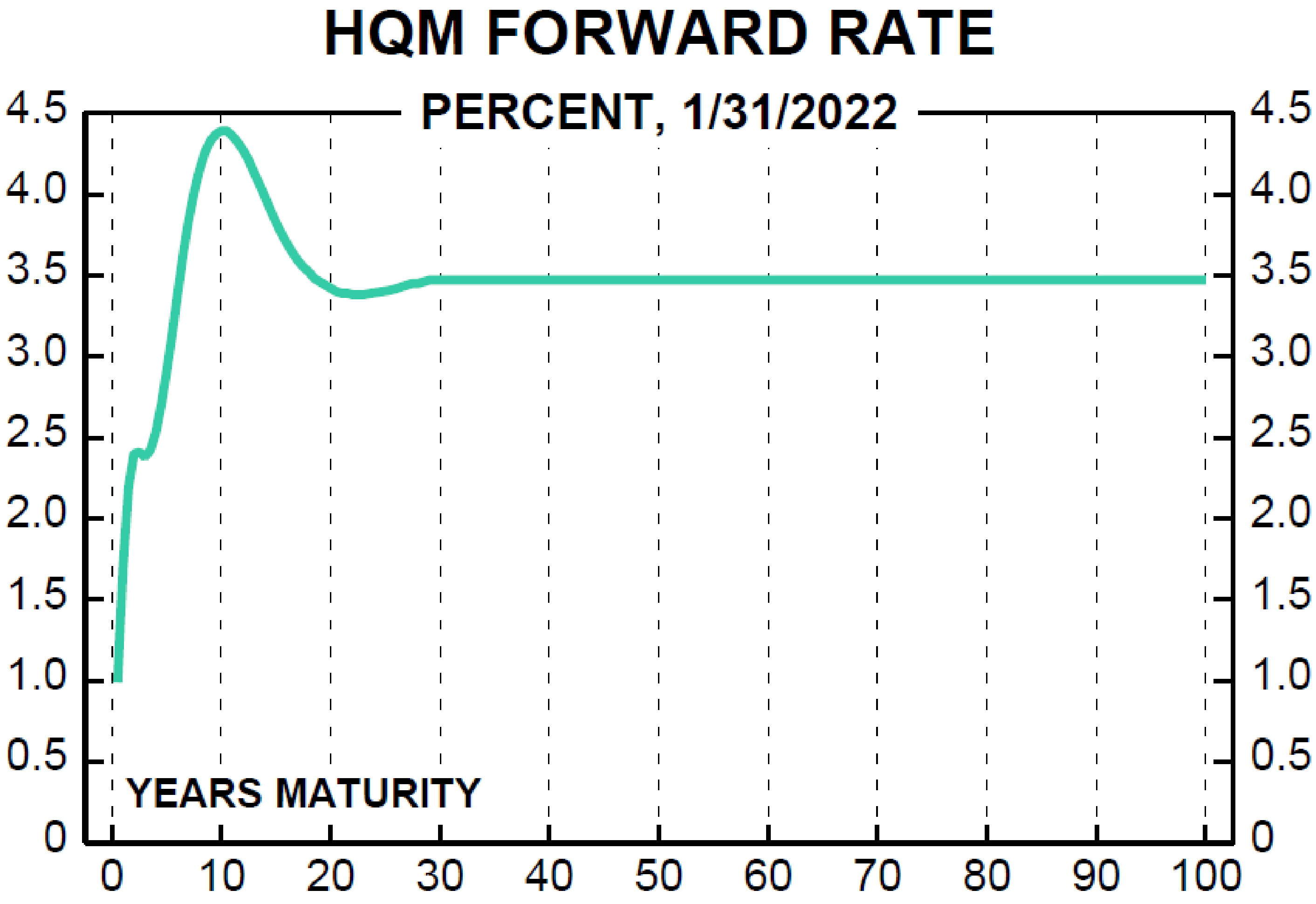
- The forward rate has a hump around 10 years maturity that is typical and shows that markets see a significant amount of risk at the point when trading transitions from medium term to long term.

- The forward rate flattens and settles down to the long-term forward rate of 3.47 percent at 30 years maturity.

HQM Discount Function, January 31, 2022



HQM Forward Rate, January 31, 2022

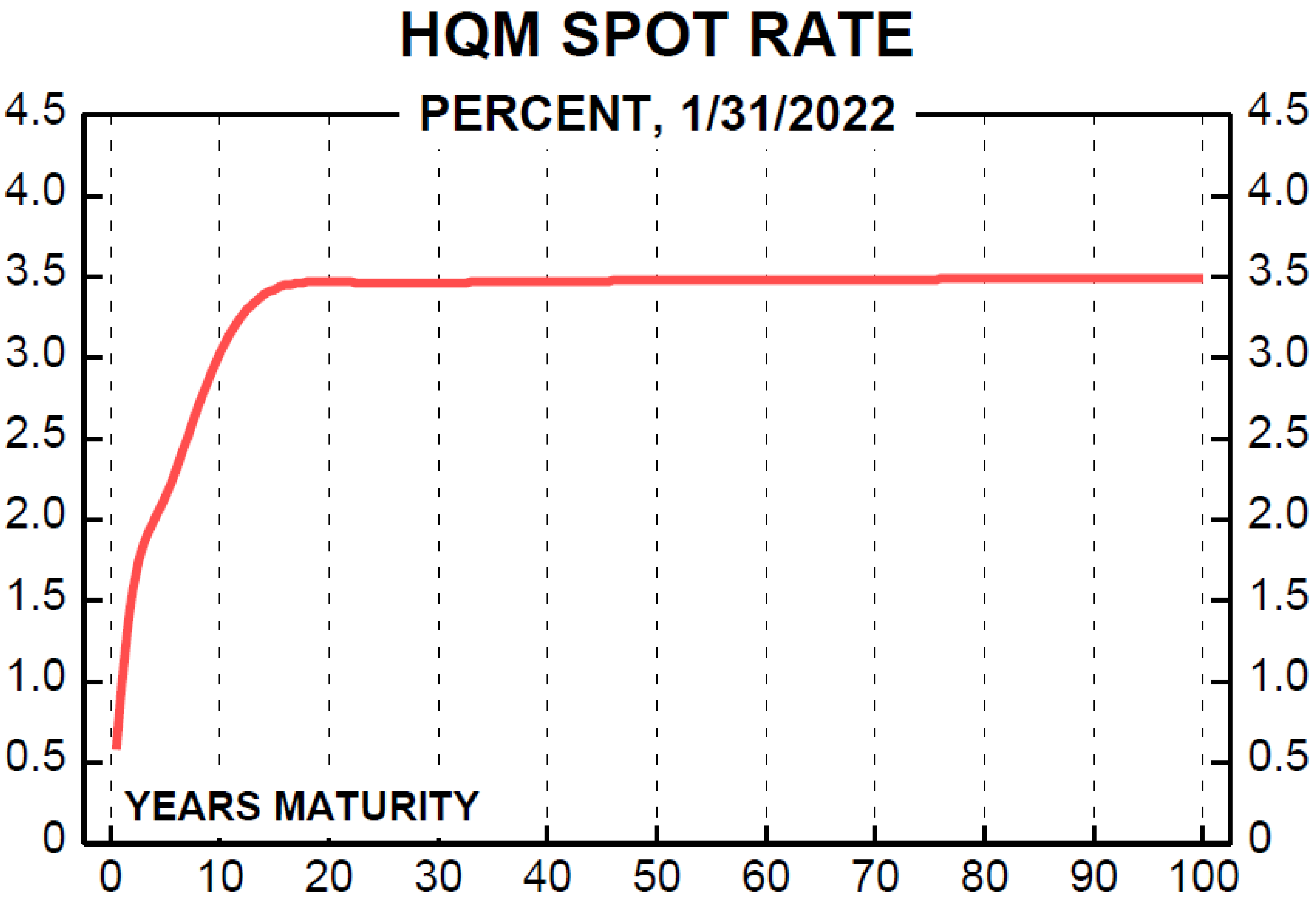


HQM Spot Rate, January 31, 2022

- The next slide shows the spot rate from the HQM yield curve for the last day of January projected out through 100 years maturity. The spot rate for this day mostly rises but has a small hump around 20 years maturity as yield curves often do.

- The spot rate starts at 0.58 percent at ½-year maturity, moves up to 3.03 percent at 10 years maturity and 3.46 percent at 30 years maturity, and then gently rises throughout the projection range reaching 3.49 percent at 100 years maturity.

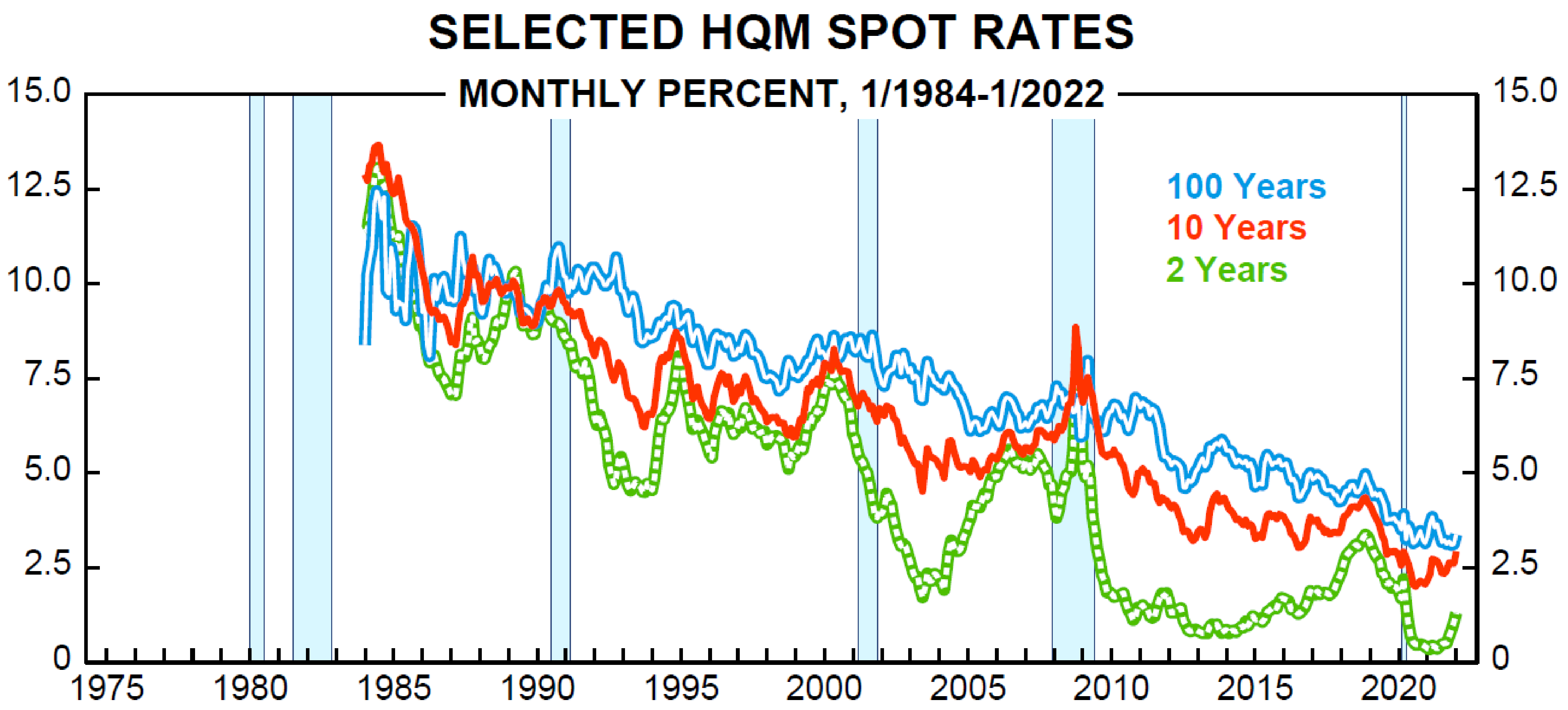
HQM Spot Rate, January 31, 2022



HQM Spot Rates over Time

- The next slide shows the monthly average spot rate for the HQM yield curve at maturities 2, 10, and 100 years from January 1984 through January 2022.
- The three spot rates tend to bunch up or even invert before recessions. Rates have been low recently but are starting to turn up.

HQM Spot Rates over Time



The Treasury Yield Curve Suite

- After the HQM yield curve, Economic Policy was requested by users to extend the HQM methodology to Treasury securities because other Treasury yield data were not sufficient for their applications.

- The result was the Treasury Nominal Coupon-Issue (TNC) Yield Curve followed by the Treasury Real Coupon-Issue (TRC) Yield Curve for Treasury Inflation-Protected Securities (TIPS) and the Treasury Breakeven Inflation (TBI) Curve combining TNC and TRC to get breakeven inflation rates.

- The TNC yield curve starts in 1976 and the TRC and TBI curves start in 2003 when the TIPS market was sufficiently developed. Both TNC and TRC use all coupon issues, and they provide the most extensive available Treasury yield curve dataset.

The TNC Yield Curve

- The TNC Yield Curve uses the same methodology as HQM, including the same maturity ranges and the same projection technique out through 100 years maturity, and provides the full range of spot rates and other results.

- The primary difference between HQM and TNC is that the TNC yield curve uses different regression variables. These variables enable the TNC curve to include all coupon issues, even those with special features that other yield curves exclude.

- The two HQM regression variables for bond ratings are omitted because Treasury securities are of the highest quality and are risk free.

- The TNC methodology includes a hump variable that picks up additional movement in the final maturity range 15 to 30 years. In particular, this variable picks up the hump in yields frequently seen around 20 years maturity that stems from lack of liquidity at 20 compared to 10 or 30 years. The hump variable can also pick up drops in yields sometimes seen around 10 years maturity.

The TNC Yield Curve, continued

- The TNC yield curve also includes dummy variables for on-the-run and first off-the-run securities. The dummies measure the effects on price of on-the-run characteristics.

- Going back historically, the TNC yield curve includes callable bonds with a regression variable to pick up their price features. Other yield curve approaches omit callables, but their omission leaves out a substantial amount of historical data especially at long maturities for the early period 1976-1984. Callable Treasuries no longer exist.

- Also historically, the TNC yield curve includes estate tax anticipation bonds with a regression variable. Commonly called flower bonds, these bonds could be redeemed at par for payment of estate taxes. Other yield curve approaches also omit flower bonds. Flower bonds don’t exist anymore.

The TRC Yield Curve

- The TRC Yield Curve is in real terms and includes all TIPS. The curve starts in 2003.
- The TRC yield curve also uses the same methodology as HQM including projection out through 100 years maturity. The regression variable is the hump variable as for TNC.

The TBI Curve

- The TBI Curve combines the TNC and TRC yield curves to derive breakeven inflation. This curve also starts in 2003.

- The TBI methodology for computing breakeven inflation is more accurate than typically seen: TBI rates use off-the-run TNC rates which are not affected by on-the-run influences and the breakeven formula is expanded correctly.

- But the most important difference is that TBI rates use TNC and TRC spot rates. Other computations typically use yields. Spot rates are correct because the logic of breakeven inflation is to determine the inflation rate that provides equal TNC and TRC returns from one point in time to another, and spot rates are the way to measure such returns.

- It appears that some of the apparent inaccuracies of breakeven inflation as an indicator of expected inflation are significantly reduced when the computation is done accurately.

TNC Par Yield, January 31, 2022

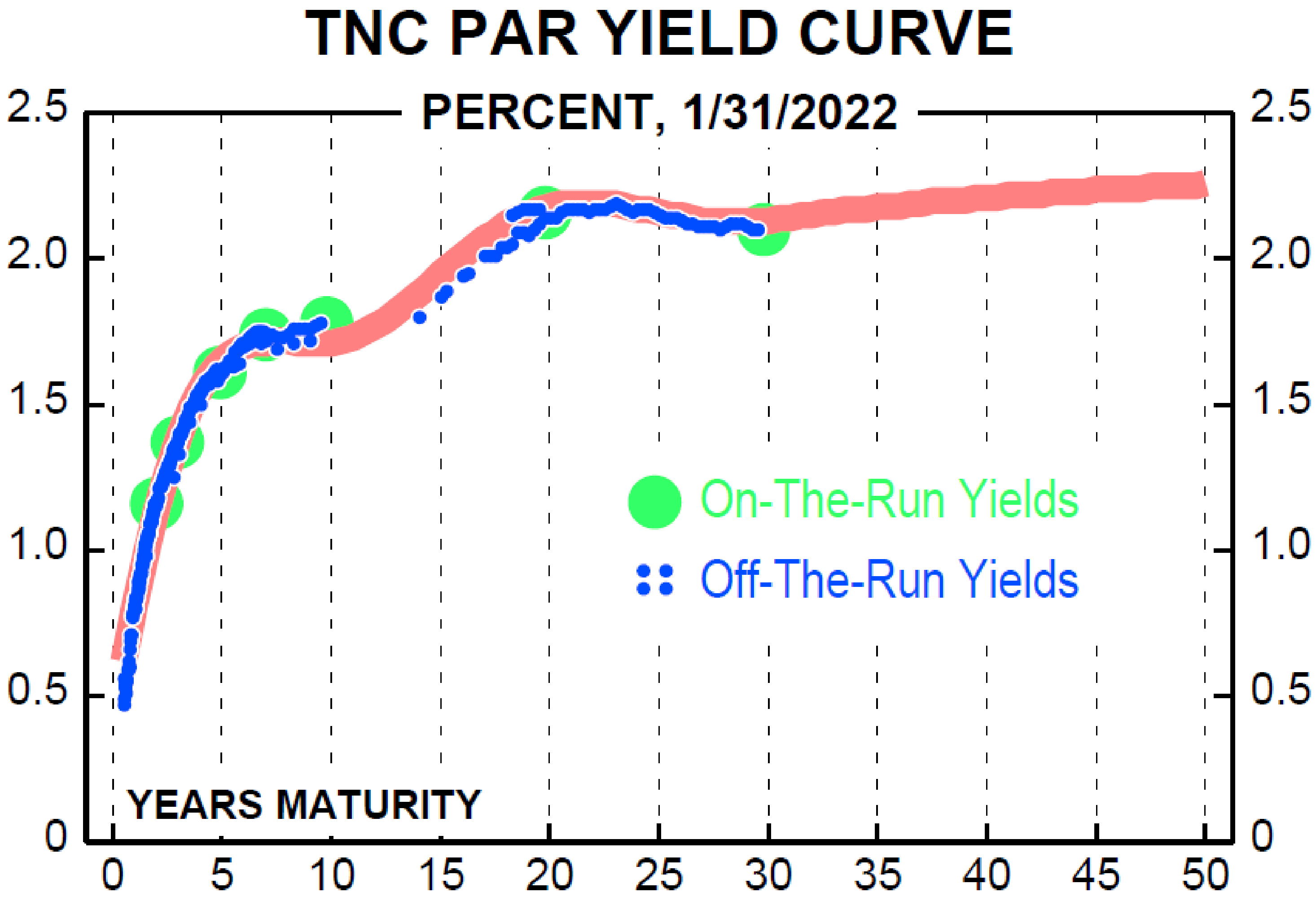
- The next slide shows the TNC par yield curve for the last day of January out through 50 years maturity and the yields for the individual securities. On this date there are 7 on-the-run securities at the terms currently being issued of 2,3,5,7,10,20,30 years and 288 off-the-run securities for a total of 295 securities.

- The curve has two humps, the first around 7 years maturity and the second around 21 years maturity. These humps enable the curve to track well the actual yields.

- On-the-run yields are near off-the-run, which hasn't necessarily been true in the past.

- This yield curve starts at 0.62 percent at the lowest maturity of ½ year, reaches 1.71 percent at 10 years maturity and 2.13 percent at 30 years maturity, then rises throughout the projection range reaching 2.26 percent at 50 years maturity and 2.33 percent at 100 years maturity.

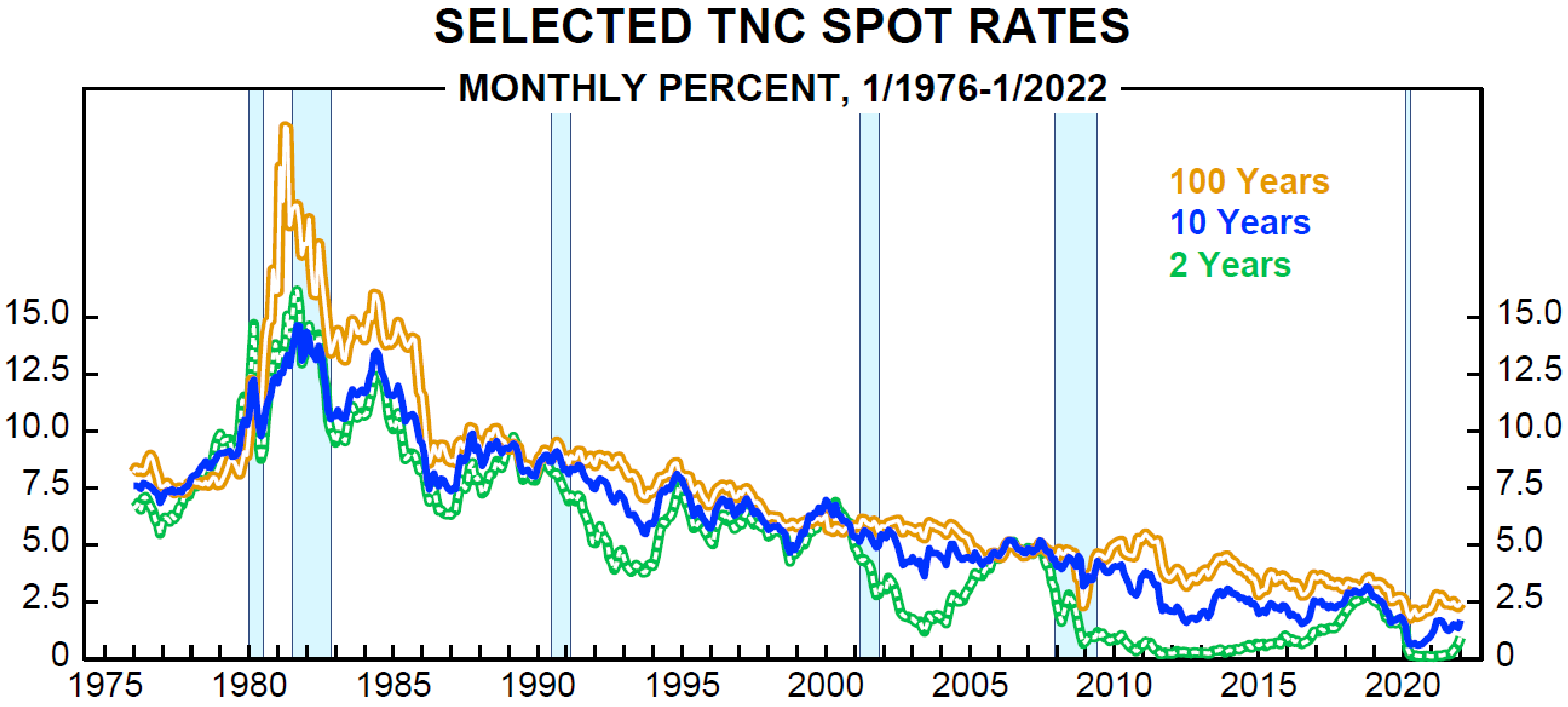
TNC Par Yield, January 31, 2022



TNC Spot Rates Over Time

- The next slide shows the monthly average spot rate for the TNC yield curve at maturities 2, 10, and 100 years from January 1976 through January 2022.
- Similar to the HQM yield curve in slide 29, the three spot rates tend to bunch up or even invert before recessions.
- The very high 100-year rates in the early 1980s reflect the high inflation at that time and the fear that inflation would continue to accelerate. Continued acceleration would require projected or hypothetical long-term nominal returns beyond 30 years maturity to rise accordingly, and the yield curve projection accurately captures this market expectation.

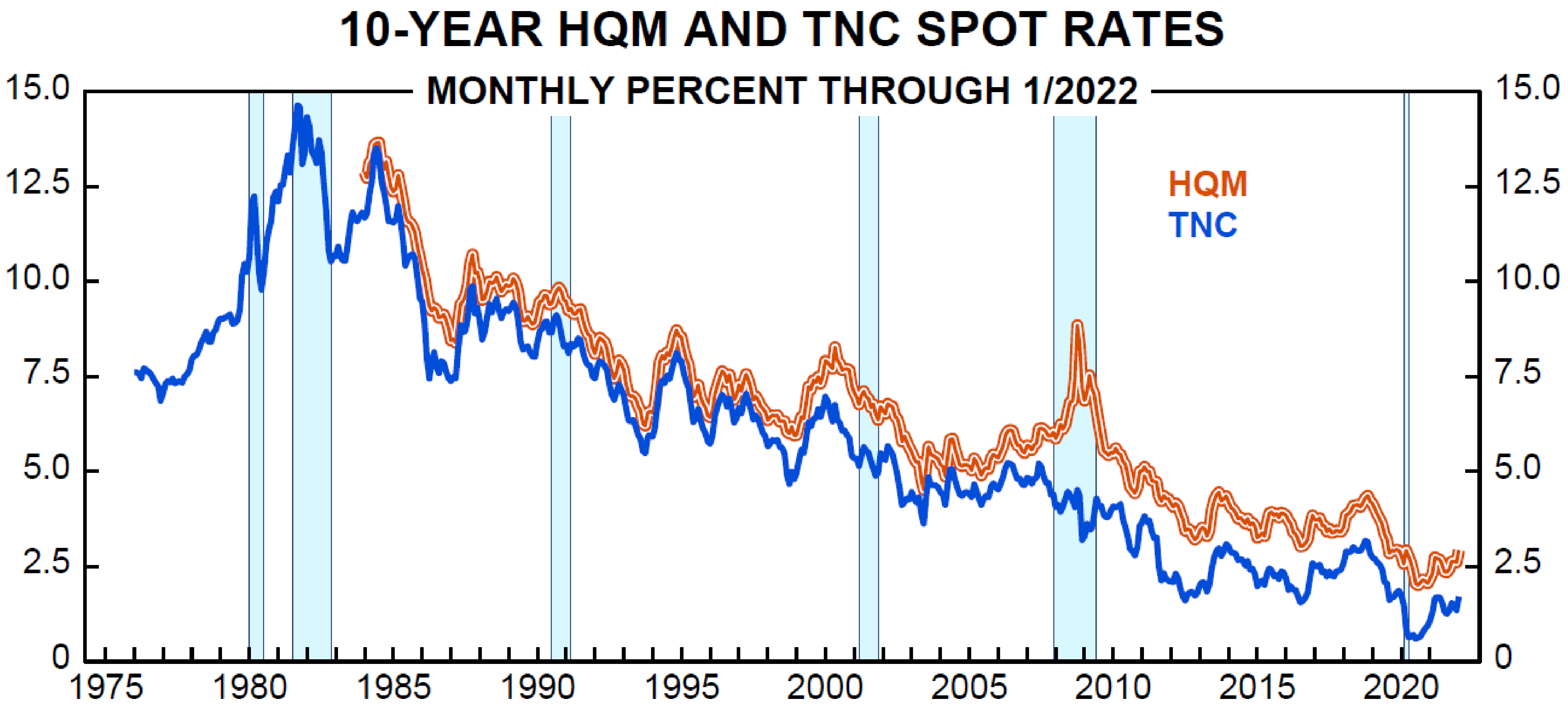
TNC Spot Rates Over Time



10-Year HQM and TNC Spot Rates Over Time

- The next slide compares historical HQM and TNC 10-year spot rates.
- The two rates mostly move together. However, in the 2007-2009 recession they moved in opposite directions because there was a flight to the safety of Treasuries relative to corporate bonds.
- The spread between the two was 122 basis points in February 2022, 133 basis on average for the last 10 years, and 113 basis points on average back through 1984.

10-Year HQM and TNC Spot Rates Over Time



Data on the Main Treasury Website

The yield curve data and more documentation can be found on the Main Treasury website. Go to Treasury.gov, choose DATA at the top, and at the bottom left the HQM yield curve can be found on the page “Corporate Bond Yield Curve and the Treasury yield curves can be found on the page “Treasury Coupon Issues”.